

Tonal, Atonal and Microtonal Pitch-Class Categories

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Abstract. This paper reviews and generalizes Pitch-Class Set Theory using Group Theory (groups acting on pc-sets) and Category Theory, which provides methods for mapping the structure of a n -tone system onto another m -tone system. This paper also suggests a new implementation approach that represents pitch-class sets as bit-sequences, which are equivalent to integer values. Forte's *best normal order* is shown to be equivalent to the smallest integer, among the cyclic permuted pc-sets. Further, transposition of pc-sets is shown to be equivalent to bit-shifts; and their inversion, to bit-reversal. The tonal (diatonic) pc-category is presented as a subset of the atonal (12-tone) pc-category, which, similarly, can also be contained in a microtonal pc-category. Functors between those categories present properties that preserve relationships while still using the same operations: tonal relationships are preserved even though atonal music operations, such as transposition and inversion are applied, allowing motives to be mapped into different modes, scales, or even microtonal scales. The appendix offers an implementation of this new approach to calculate and represent pc-sets with an arbitrary number of pitch-classes.

1 Introduction

Pitch-classes are generally defined as musical notes assigned to fixed integer values given by the remainder of octave division by 12, as depicted in the top row of figure 1:

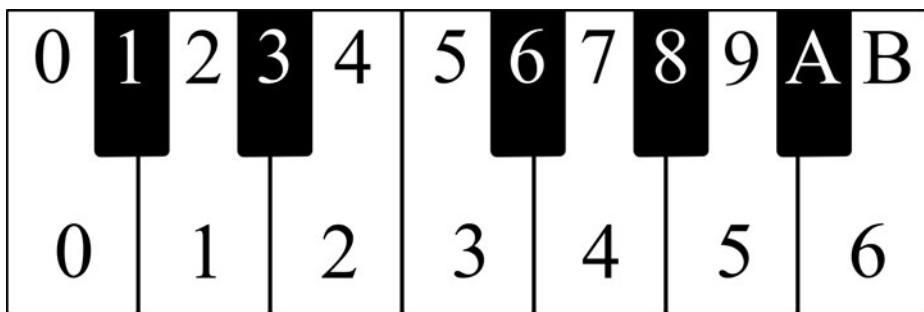


Fig. 1. Two representations of pitch-classes: 12-tone (top) and a diatonic major subset (bottom)

This paper suggests different representations of pitch-classes depending on their context, such as the subset of white piano keys (bottom row of figure 1), which could be seen as a diatonic major scale starting at the pitch C.

Pitch-classes are generally represented as mathematical sets. Nevertheless, agreements on a standard notation for sets of pitch-classes have not yet been reached. This paper uses the following notation conventions: $[2, 0, (n-1), \dots]_n$ representing unordered sets, and $[0123\dots(n-1)]_n$ meaning partially ordered sets. In both cases n represents the total number of pitch-classes in context, therefore not the cardinality of proper subsets. Whenever n is not given, it follows the traditional context of 12 pitch-classes. For the sake of clarity, nomenclature related to pitch-class set theory is henceforth italicized.

Representation of notes as pitch-classes assumes octave and enharmonic equivalences. Within tonal music, however, enharmonic equivalence is not assumed: besides rules of voice-leading, pitch spelling depends on musical key or tonal center, according to which pitches are contextualized. Further, octave equivalence is not assumed in case of extended chords such as ninth or eleventh chords that are common in jazz and other musical styles.

2 Applying Pitch-Class Set Theory on Sets with Cardinality (Pitch-Classes) Other Than 12

The term *Pitch-Class Set* (pc-set) as used in musical analysis and composition was introduced by Babbitt (Forte 1973). Pitch-Class Set theory and terminology was developed by Forte (1973), Rahn (1980), and Straus (1990), among others. Equivalence of pc-sets had been extensively discussed. Morris (1982) compared four distinct approaches to partitioning the *universe*¹ (power set) of pc-sets.

The main operations on pc-sets are *transposition* and *inversion*. Forte defined the terminology for classification of pitch-class sets: pc-sets can be reduced to a *normal order*, in which pc-sets are ordered according to their pitch-classes. *Normal orders* are cyclic permutations (*transpositions*) of the *best normal order*, which is the permutation that presents the smallest, ordered intervals, starting at pitch class zero. Finally, the *prime form* is the ‘best’ (smallest) between the *best normal order* of a given pc-set and the *best normal order* of its inversion.

Transposition equivalence of a pc-set within the *universe* (power set) of a set of twelve pitch-classes is isomorphic to chromatic transposition, in which all (chromatic) intervals between pitch-classes are maintained. Considering, however, a power set of a set of seven pitch-classes that form a diatonic major scale, diatonic triads can be seen as circular permutations of the same pc-set₇, preserving their tonal relationships, while still being a result of the same operations used on pc-set₁₂:

¹ “U*, the universal set of 4096 pcsets, has been partitioned into collections of “equivalent” sets in a number of different ways. In the four systems we shall examine, U* is initially divided into collections of sets of equal size. Pcset equivalence is further defined by interval-class content or by identity under certain twelve-tone operators, which may be any combination of *Tn*, *I*, and/or *M*.” (Morris 1982, 102) The ‘universal set’ described by Morris is a power set. Classification of pc-sets in this paper is based on group action and is in accordance with Forte (1973). The discussion on classification regarding interval classes is beyond the scope of this paper.