A Classification of Seven Tone Scales

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Abstract: In this work we provide an original approach to the classification of seven tone scales from the view point of composition and improvization. There are a total of 66 seven tone (heptatonic) scale formulas for equitempered systems out of which we eliminate 34 of them in view of the principles of tonal harmony and classify the remaining 32 scales into seven groups which are associated with the seven modes of the diatonic major scale. Such a classification is claimed to provide a perspective in understanding harmonic progressions and improvizational techniques for both the educator and the composer. We also provide a number of musical examples that demonstrate our methodology.

1. Introduction

Classification is one of the most fundamental scientific methods in attempts to describe and analyze any concept or problem under consideration. With regard to musical scales the history of such attempts yield back to the era of ancient Greeks. In literature a systematic investigation of "synthetic scales", i.e., scales formed by raising and lowering the pitches of the tones of the *natural* diatonic major scale, is generally attributed to the book of the great composer Busoni [1], which later has been improved and cathegorized in a paper by Mason [2]. In [2] a total of 1254 seven tone "Busoni scales" have been enlisted under the tonal limitation that the accidential of each tone is restricted to (𝄫, ♭, ♮,

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#,×). Such a high number in Busoni scales arises when an enharmonic mapping onto the twelve tones of the equitempered system is avoided (i.e., when F♭ is not assumed equal to E and so forth). However, Busoni scales have been critisized in literature as early as 1929, around 18 years after they appeared, by Barbour [3]. In his related works [3,4] Barbour discusses the synthesis of seven tone scales in *equal temparament* within the range of tonal harmony and critisizes that the enumeration of Busoni scales are not supported by any theoretical background that interconnects them. In that context Busoni addresses a total of 66 seven tone cyclic scale formulas, which is also the starting point of the current investigation.

2. The 66 Seven Tone Scale Formulas in Equal Temparament

Based on the principle of cyclic permutation the mentioned 66 seven tone scale formulas are derived and grouped into seven classes as depicted in Table 1.

Class	Set of Semitone Intervals	No. of Different Cyclic Formulas	No. of Modes
I	{1,1,2,2,2,2,2}	(7-1)!/(2!*5!)=3	3*7=21
II	{1,1,1,2,2,2,3}	(7-1)!/(3!*3!)=20	20*7=140
III	{1,1,1,1,2,3,3}	(7-1)!/(4!*2!)=15	15*7=105
IV	{1,1,1,1,2,2,4}	(7-1)!/(4!*2!)=15	15*7=105
V	{1,1,1,1,1,3,4}	(7-1)!/5!=6	6*7=42
VI	{1,1,1,1,1,2,5}	(7-1)!/5!=6	6*7=42

Table 1. The seven tone scales grouped into seven classes

It is observed that in case of equal temparament the 1254 modes of Busoni in [2] reduce into 66*7=462 modes. Certainly, this is still a huge number to benefit effectively for compositional purposes in any tonality. Therefore it should be expected that further restrictions are imposed in accord with the principles of tonal harmony depending on the purpose of a composer.

In his 1949 paper [4] Barbour handles this subject and critisizes Delezenne [5], Gandillot [6], Hatherly [7] and Helmholtz [8] for avoiding 31 of 66 scales due to insistance in these references that any scale that "works" should involve at least two perfect fifths. In that context Barbour draws attention to the melodic varieties offered by the rest of those scales, specially in connection with ragas of Indian music.

One should also mention to the book by Slonimsky [9, pp.137-154] first published in 1947 where a total of 54 out of 224 modes, though unsystematically, are enlisted as constructed over the note C (i.e., a "C scale") also accompanied by 4 note chord voicings.

At this point we wish to introduce our own perspective on this set of 66 seven tone scales. We consider a subset of these scales for which all mode formulas can be constructed as a C scale under the natural and unique restriction that accidentals on each tone is always limited to (𝄫, ♭, ♮, #, 𝄪). In other words, we describe a scale formula that "works" as one

for which *all* seven modes permit a C scale limited to the mentioned accidential restrictions. From this perspective we are left with only 32 out of the 66 scales. They comprise all 23 scales of Classes I and II and 9 out of 15 scales of Class III. The 6 scale formulas of Class III that are eliminated are enlisted in Table 2. It is an easy task for any reader to verify that *at least* one mode of every cyclic scale formula in Table 2 does not permit a C scale.

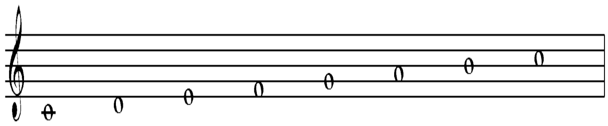
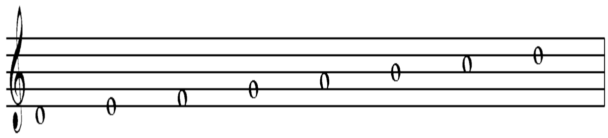
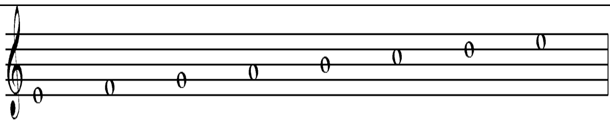
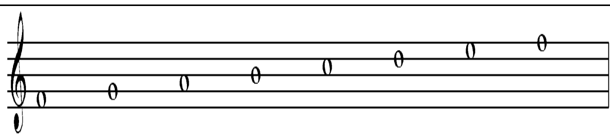
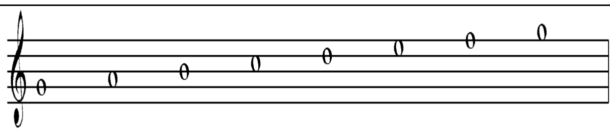
3-1-1-1-1-2-3	3-3-1-1-1-2-1	3-3-2-1-1-1-1
3-3-1-1-2-1-1	3-3-1-2-1-1-1	3-2-3-1-1-1-1

Table 2. The eliminated 6 cyclic scale formulas of Class III

To support our claim we introduce and provide the melodic connection inbetween these 32 scale formulas in the following section.

3. A Classification of the 32 Seven Tone Scale Formulas That "Work"

Our perspective in a classification of any scale that "works" is that its structure should be connected to any of the well known seven modes of the natural diatonic major scale depicted in Table 3.

Mode Number	Mode Formula	Mode Name	Treble Clef Representation
1	(221)+2+(221)	Ionian	
2	(212)+2+(212)	Dorian	
3	(122)+2+(122)	Phrygian	
4	(222)+1+(221)	Lydian	
5	(221)+2+(212)	Mixolydian	

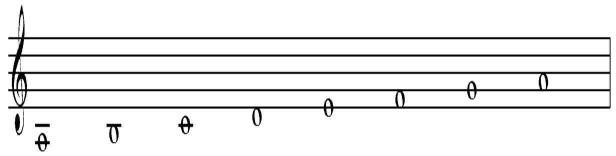
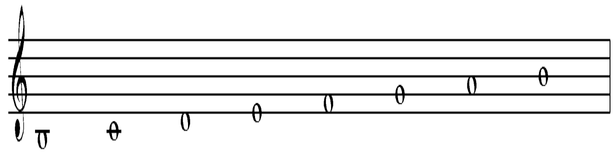
6	(212)+2+(122)	Aeolian	
7	(122)+1+(222)	Locrian	

Table 3. The natural diatonic major scale as the generator of all synthetic scales

In Figure 1 we illustrate the mathematical distribution of the 31 synthetic scales under the seven groups represented by the modes of the generator diatonic major scale.

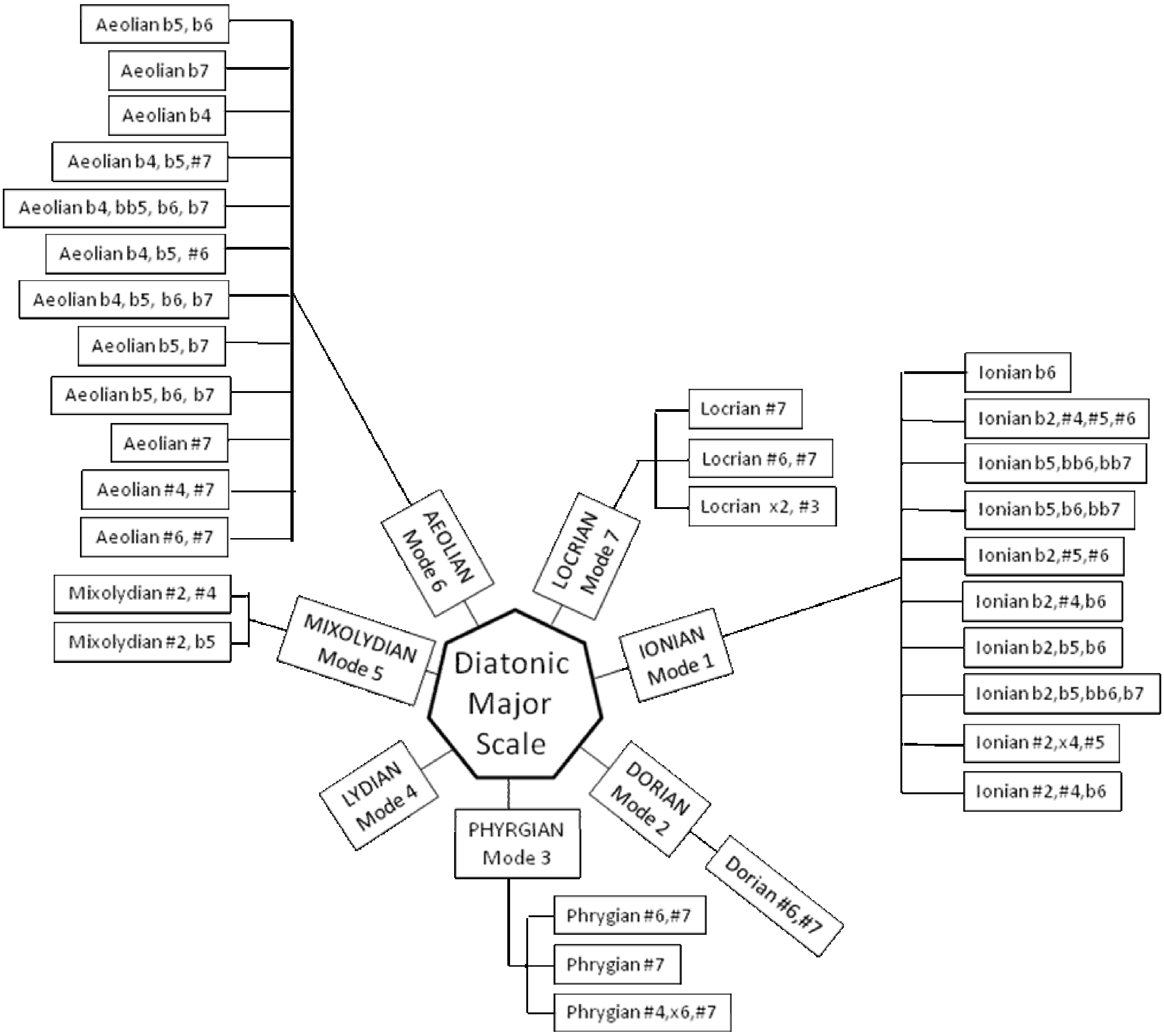


Figure 1. The mathematical family tree of seven tone C scales

The details of the idea behind this grouping is as follows: Since the Ionian is a major scale (with a major third), all root scale formulas with a major third are located under this group. Similarly, since the Aeolian is a minor scale (with a minor third), all root scale formulas with a minor third are located thereunder. It is seen that a total of 23 scale families out of 32 are grouped under the natural major and minor scales. As for the groups under Dorian, Phrygian, Mixolydian and Locrian scales a similar idea is employed where stronger tonal relations can be observed since in most cases the whole lower tetrachord structure keeps unchanged. In this representation it is seen that none of the synthetic scales is related to the Lydian mode. It stems from the fact that the lower tetrachord of Lydian mode ranges on a 6 semitone interval, while the rest of the mode range on a 5 semitone interval. We shall emphasize on this issue in more detail in Section 4 where we introduce a classification of tetrachords.

And in the determination of the root scale formulas out of seven modes indicated by each one of the 31 cyclic formula, we have picked those for which the *simultaneous* appearance of tones with flat and sharp accidentials is at minimum in view of tonal harmony. These principles for melodic connections are illustrated in the following 6 tables representing the 6 branches of the family tree.

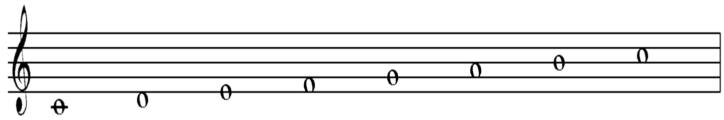
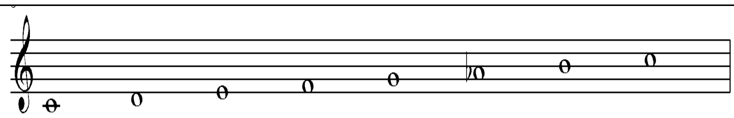
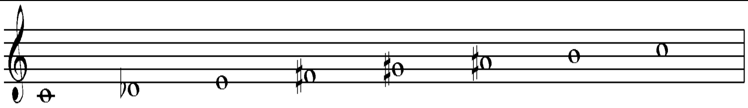
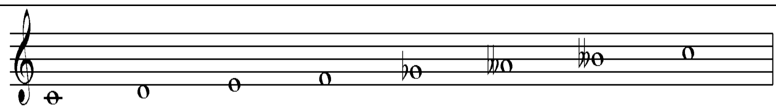
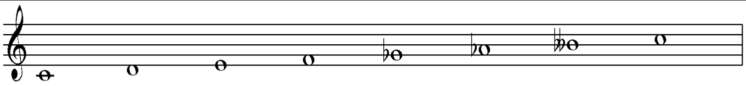
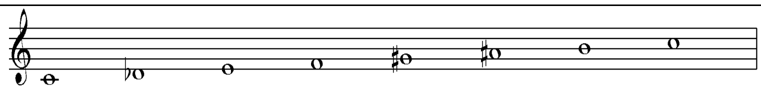
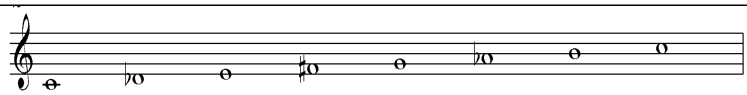
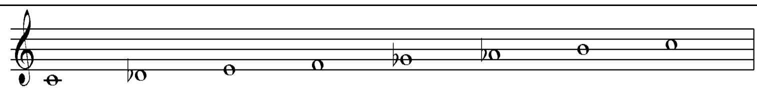
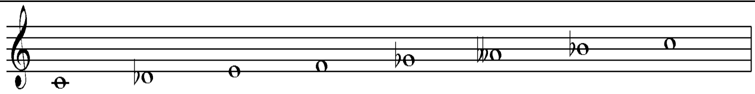
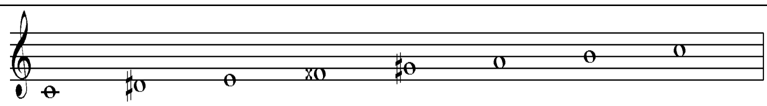
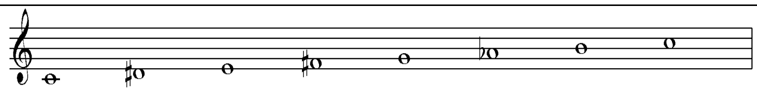
Scale Name	Scale Formula	Treble Clef Representation
Ionian (Generator Mode 1)	(221)+2+(221)	
Ionian ♭6	(221)+2+(131)	
Ionian ♭2,#4,#5,#6	(132)+2+(211)	
Ionian ♭5,𝄫6,𝄫7	(221)+1+(123)	
Ionian ♭5,♭6,𝄫7	(221)+1+(213)	
Ionian ♭2,#5,#6	(131)+3+(211)	
Ionian ♭2,#4,♭6	(132)+1+(131)	
Ionian ♭2,♭5,♭6	(131)+1+(231)	
Ionian ♭2,♭5,𝄫6,♭7	(131)+1+(132)	
Ionian #2,𝄪4,#5	(313)+1+(121)	
Ionian #2,#4,♭6	(312)+1+(131)	

Table 4. The scales generated by Ionian Mode

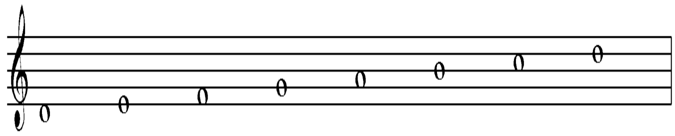
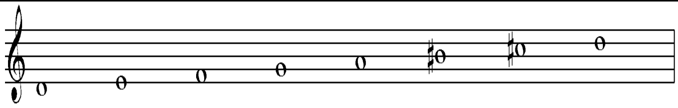
Scale Name	Scale Formula	Treble Clef Representation
Dorian (Generator Mode 2)	(212)+2+(212)	
Dorian #6,#7	(212)+2+(311)	

Table 5. The scale generated by Dorian Mode

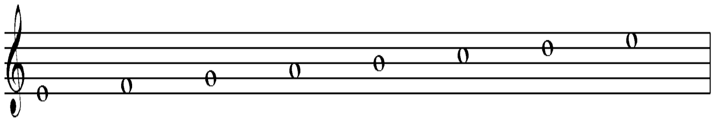
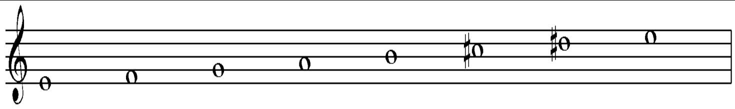
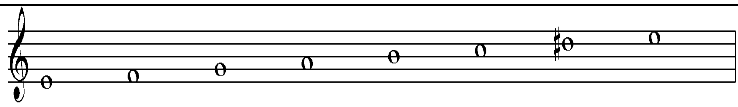
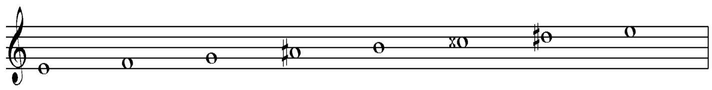
Scale Name	Scale Formula	Treble Clef Representation
Phrygian (Generator Mode 3)	(122)+2+(122)	
Phrygian #6,#7	(122)+2+(221)	
Phrygian #7	(122)+2+(131)	
Phrygian #4,x6,#7	(123)+1+(311)	

Table 6. The scales generated by Phrygian Mode

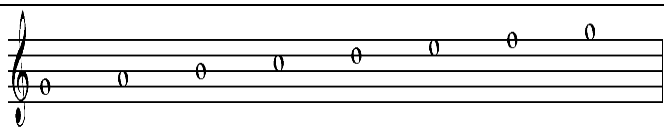
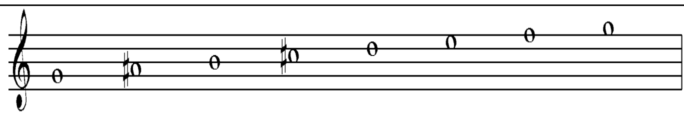
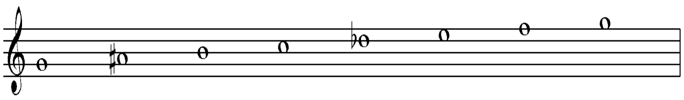
Scale Name	Scale Formula	Treble Clef Representation
Mixolydian (Generator Mode 5)	(221)+2+(212)	
Mixolydian #2,#4	(312)+1+(212)	
Mixolydian #2,♭5	(311)+1+(312)	

Table 7. The scales generated by Mixolydian Mode

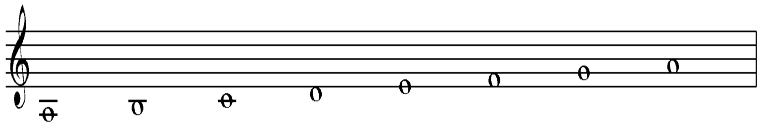
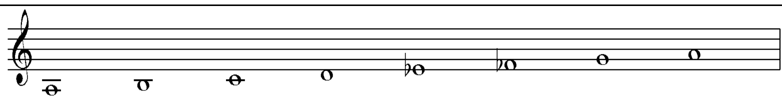
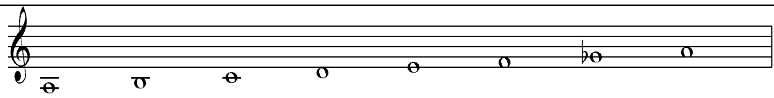
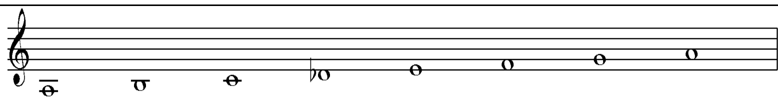
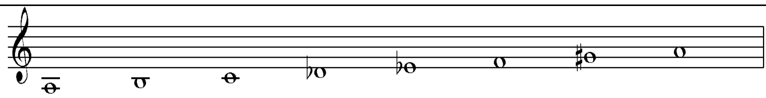
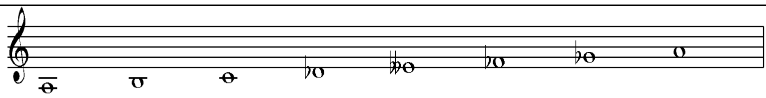
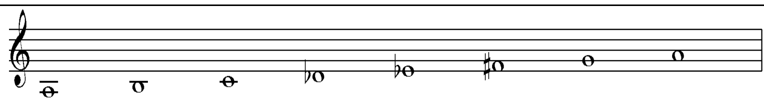
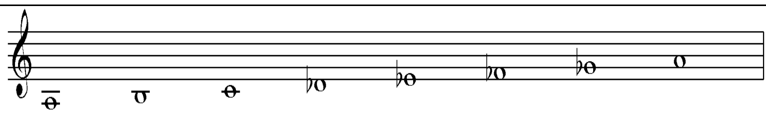
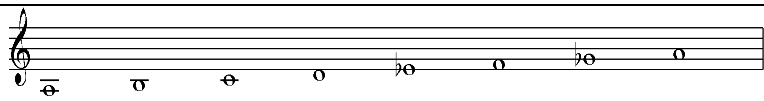
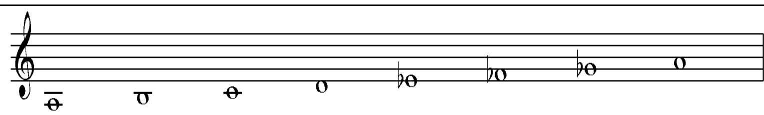
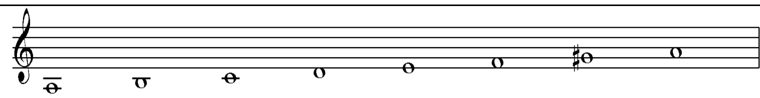
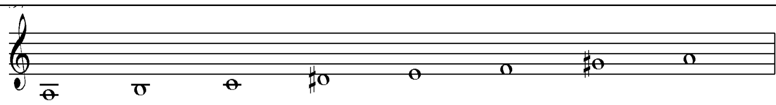
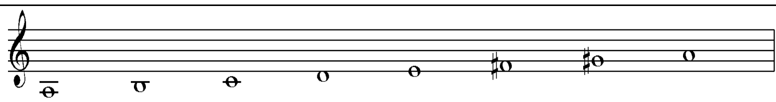
Scale Name	Scale Formula	Treble Clef Representation
Aeolian (Generator Mode 6)	(212)+2+(122)	
Aeolian ♭5,♭6	(212)+1+(132)	
Aeolian ♭7	(212)+2+(113)	
Aeolian ♭4	(211)+3+(122)	
Aeolian ♭4,♭5,#7	(211)+2+(231)	
Aeolian ♭4,𝄫5,♭6,♭7	(211)+1+(223)	
Aeolian ♭4,♭5,#6	(211)+2+(312)	
Aeolian ♭4,♭5,♭6,♭7	(211)+2+(123)	
Aeolian ♭5,♭7	(212)+1+(213)	
Aeolian ♭5,♭6,♭7	(212)+1+(123)	
Aeolian #7	(212)+2+(131)	
Aeolian #4,#7	(213)+1+(131)	
Aeolian #6,#7	(212)+2+(221)	

Table 8. Scales generated by Aeolian Mode

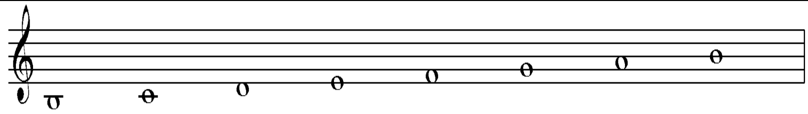
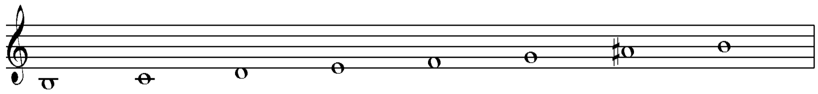
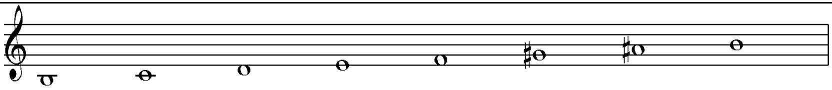
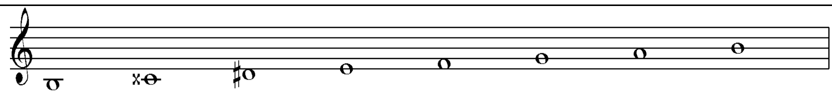
Scale Name	Scale Formula	Treble Clef Representation
Locrian (Generator Mode 7)	(122)+1+(222)	
Locrian #7	(122)+1+(231)	
Locrian #6,#7	(122)+1+(321)	
Locrian ×2,#3	(311)+1+(222)	

Table 9. Scales generated by Locrian Mode

The distribution of these 32 root scale formulas into Classes I, II and III are displayed in Tables 10-12.

Scale	Root Formula
Diatonic Major	(221)+2+(221)
Phrygian #6,#7	(122)+2+(221)
Aeolian #6,#7	(212)+2+(221)

Table 10. Scale families falling into Class I

Scale	Root Formula
Ionian ♭6	(221)+2+(131)
Ionian ♭2,#4,#5,#6	(132)+2+(211)
Ionian ♭5,𝄫6,𝄫7	(221)+1+(123)
Ionian ♭5,♭6,𝄫7	(221)+1+(213)
Dorian #6,#7	(212)+2+(311)
Phrygian #7	(122)+2+(131)
Mixolydian #2,#4	(312)+1+(212)
Aeolian ♭5,♭6	(212)+1+(132)

Aeolian ♭7	(212)+2+(113)
Aeolian ♭4	(211)+3+(122)
Aeolian ♭4,♭5,#7	(211)+2+(231)
Aeolian ♭4,𝄫5,♭6,♭7	(211)+1+(223)
Aeolian ♭4,♭5,#6	(211)+2+(312)
Aeolian ♭4,♭5,♭6,♭7	(211)+2+(123)
Aeolian ♭5,♭7	(212)+1+(213)
Aeolian ♭5,♭6,♭7	(212)+1+(123)
Aeolian #7	(212)+2+(131)
Locrian #7	(122)+1+(231)
Locrian #6,#7	(122)+1+(321)
Locrian 𝄪2,#3	(311)+1+(222)

Table 11. Scale families falling into Class II

Scale	Root Formula
Ionian ♭2,#5,#6	(131)+3+(211)
Ionian ♭2,#4,♭6	(132)+1+(131)
Ionian ♭2,♭5,♭6	(131)+1+(231)
Ionian ♭2,♭5,𝄫6,♭7	(131)+1+(132)
Ionian #2, 𝄪4,#5	(313)+1+(121)
Ionian #2,#4,♭6	(312)+1+(131)
Phrygian #4, 𝄪6,#7	(123)+1+(311)
Mixolydian #2,♭5	(311)+1+(312)
Aeolian #4,#7	(213)+1+(131)

Table 12. Scale families falling into Class III

4. A Classification of Tetrachords

For a deeper insight on the melodic connections between the 32 cyclic formulas we also introduce symbolic representations for the tetrachords constituting the scales through the notation XT_Y, where the letter T stands for the initial of tetrachord; X denotes the total number of semitones in a tetrachord; and the subscript Y is the index parameter differentiating between the tetrachords having the same X value. With all these in mind in Tables 13 and 14 we provide a list of all possible natural ($X = 5,6$) and altered ($X = 3,4,7$) tetrachord formulas associated with the 32 scales under consideration.

Set of Semitones	Tetrachord Formula	Tetrachord Symbol
{1,2,2}	1-2-2	$5T_1$
{1,1,3}	2-1-2	$5T_2$
	2-2-1	$5T_3$
	1-1-3	$5T_4$
{2,2,2}	1-3-1	$5T_5$
	3-1-1	$5T_6$
	2-2-2	$6T_1$
{1,2,3}	1-2-3	$6T_2$
	1-3-2	$6T_3$
	2-1-3	$6T_4$
	2-3-1	$6T_5$
	3-1-2	$6T_6$
	3-2-1	$6T_7$

Table 13. A list of natural tetrachord formulas and symbols

Set of Semitones	Tetrachord Formula	Tetrachord Symbol
{1,1,1}	1-1-1	$3T$
{1,1,2}	1-1-2	$4T_1$
	1-2-1	$4T_2$
	2-1-1	$4T_3$
{2,2,3}	2-2-3	$7T_1$
	2-3-2	$7T_2$
	3-2-2	$7T_3$
{1,3,3}	1-3-3	$7T_4$
	3-1-3	$7T_5$
	3-3-1	$7T_6$

Table 14. A list of altered tetrachord formulas and symbols

In Table 14 a notational convenience is introduced for the first tetrachord by removing the subscript which is necessarily equal to 1.

Now the mathematical structure of all 32*7=224 modes we consider can be tabulated by the 2*9=18 scale structures given in Table 15.

Order	Scale Structure	Corresponding Opposite Scale Structure
1	$3T+2+7T_Y$ or $(3T,7T_Y)$	$7T_Y+2+3T$ or $(7T_Y,3T)$
2	$3T+3+6T_Y$ or $(3T,6T_Y)$	$6T_Y+3+3T$ or $(6T_Y,3T)$
3	$3T+4+5T_Y$ or $(3T,5T_Y)$	$5T_Y+4+3T$ or $(5T_Y,3T)$
4	$4T_X+1+7T_Y$ or $(4T_X,7T_Y)$	$7T_Y+1+4T_X$ or $(7T_Y,4T_X)$
5	$4T_X+2+6T_Y$ or $(4T_X,6T_Y)$	$6T_Y+2+4T_X$ or $(6T_Y,4T_X)$
6	$4T_X+3+5T_Y$ or $(4T_X,5T_Y)$	$5T_Y+3+4T_X$ or $(5T_Y,4T_X)$
7	$4T_X+4+4T_Y$ or $(4T_X,4T_Y)$	$4T_Y+4+4T_X$ or $(4T_Y,4T_X)$
8	$5T_X+1+6T_Y$ or $(5T_X,6T_Y)$	$6T_Y+1+5T_X$ or $(6T_Y,5T_X)$
9	$5T_X+2+5T_Y$ or $(5T_X,5T_Y)$	$5T_Y+2+5T_X$ or $(5T_Y,5T_X)$

Table 15. A list of all possible scale structures for 224 modes

It should be realized that the coefficients in each structure sum up to 12. Therefore one can express the sum formula in a shorter way as in paranthesis avoiding the connector semitone, which is determined uniquely in every case.

The distribution of tetrachords into the 32 scales of Classes I, II, III are enlisted in Table 16.

Class	Tetrachords
I	$4T_1, 4T_2, 4T_3; 5T_1, 5T_2, 5T_3; 6T_1$
II	$3T; 4T_1, 4T_2, 4T_3; 5T_1, 5T_2, 5T_3, 5T_4, 5T_5, 5T_6$ $6T_1, 6T_2, 6T_3, 6T_4, 6T_5, 6T_6, 6T_7; 7T_1, 7T_2, 7T_3$
III	$3T; 4T_1, 4T_2, 4T_3; 5T_4, 5T_5, 5T_6; 6T_1; 7T_4\ 7T_5, 7T_6$

Table 16. The distribution of tetrachords into scales of Classes I, II, III

This new notation provides an opportunity to observe the mathematical symmetries and correlations in the tetrachord structure of the 32 cyclic scale formulas. In Table 17 we display the case for the diatonic major scale

Mode Number	Mode Name	Mode Symbol
1	Ionian	$(5T_3, 5T_3)$
2	Dorian	$(5T_2, 5T_2)$
3	Phrygian	$(5T_1, 5T_1)$
4	Lydian	$(6T_1, 5T_3)$
5	Mixolydian	$(5T_3, 5T_2)$
6	Aeolian	$(5T_2, 5T_1)$
7	Locrian	$(5T_1, 6T_1)$

Table 17. The tetrachord structure of the diatonic major scale

As seen in Table 17 the lower tetrachord of Lydian mode is a $6T_1$, while it is of type $5T_X$ for the rest of the modes. The interval for $6T_1$ is an augmented fourth (tritone) which is known in literature as “diabolus in musica”. There underlies two fundamental purposes that lead the evolutional period from modal to tonal music. The former is related to the lower tetrachord ($6T_1$) of Lydian mode and the latter is related to the upper tetrachords of Dorian ($5T_2$), Mixolydian ($5T_2$) and Phrygian ($5T_1$) modes. The requirement of converting (“musica ficta”) the lower tetrachord of Lydian mode to a $5T_3$ is due to the concerns to avoid the “diabolus in musica”, while that for converting the upper tetrachords of Dorian, Mixolydian and Phyrgian modes to a $5T_3$ is for generating a leading tone in a scale. It is shown that the present symbolic analysis for the tetrachords serve to display this aural fact which goes back to 13th century with a mathematical language.

One may also note that 4 among the 224 modes have a self-inverse structure as given in Table 18.

Scale and Mode	Mode Formula	Mode Symbol
Phrygian #6,#7 Scale Mode 1	(122)+2+(221)	$(5T_1, 5T_3)$
Aeolian #6,#7 Scale Mode 5 (Melodic Major)	(221) +2+(122)	$(5T_3, 5T_1)$
Diatonic Major Scale Mode 2 (Dorian)	(212) +2+(212)	$(5T_2, 5T_2)$
Aeolian #4,#7 Scale Mode 5 (Double Harmonic Major)	(131)+2+(131)	$(5T_5, 5T_5)$

Table 18. Self inverse modes

In the classification of the cyclic scale formulas constituting the mathematical family tree it is already mentioned that the structure of the lower tetrachords are taken into consideration. Now under the new notation further remarks on classification can be made over the lower tetrachords. Out of the 32 root scales 26 of them are seen to possess natural lower tetrachords, while the remaining 6 possess altered ones. Their properties are summarized in Tables 19 and 20.

Scale	Lower Tetrachord Structure	Common Properties
Dorian #6,#7	$5T_2$	The lower tetrachords coincide exactly with those of natural major or minor scales
Aeolian ♭5,♭6		
Aeolian ♭7		
Aeolian ♭5,♭7		
Aeolian ♭5,♭6,♭7		
Aeolian #7		
Aeolian #6,#7		
Ionian	$5T_3$	
Ionian ♭6		
Ionian ♭5,𝄫6,𝄫7		
Ionian ♭5,♭6,𝄫7		
Phrygian #6,#7	$5T_1$	The lower tetrachords differ from those of natural major or minor scales only by an accidental on the second note/tone
Phrygian #7		
Locrian #7		
Locrian #6,#7		
Ionian ♭2,#5,#6	$5T_5$	
Ionian ♭2,♭5,♭6		
Ionian ♭2,♭5,𝄫6,♭7		
Mixolydian #2,♭5	$5T_6$	
Locrian 𝄪2,#3		
Aeolian #4,#7	$6T_4$	The lower tetrachords differ from that of natural minor scale only by an accidental on the fourth note/tone
Phrygian #4,𝄪6,#7	$6T_2$	The lower tetrachords differ from those of natural major or minor scales only by accidentals on the second and fourth notes/tones
Ionian ♭2,#4,#5,#6	$6T_3$	
Ionian ♭2,#4,♭6		
Ionian #2,#4,♭6	$6T_6$	
Mixolydian #2,#4		

Table 19. Root scales possessing natural lower tetrachords

Scale	Lower Tetrachord Structure	Common Properties
Aeolian ♭4	$4T_3$	The lower tetrachords differ from that of natural minor scale only by an accidental on the fourth note/tone
Aeolian ♭4,♭5,#7		
Aeolian ♭4,𝄫5,♭6,♭7		
Aeolian ♭4,♭5,#6		
Aeolian ♭4,♭5,♭6,♭7		
Ionian #2,𝄪4,#5	$7T_5$	The lower tetrachords differ from that of natural major scale only by accidentals on the second and fourth notes/tones

Table 20. Root scales possessing altered lower tetrachords

5. Observations on the Scale Nomenclature

Regarding the scale nomenclature, we shall confine ourselves to the ancient Greek modes in Table 3 and leave the ethnic, cultural or regional names for much of the 224 scales and certain of the tetrachords as the subject of another work.

Of critical importance are the standard harmonic and melodic major/minor scales constructed over the Greek modes as presented in Table 21.

Scales over Greek Modes	Standard Nomenclature
Ionian ♭6	The (Ionic) Harmonic Major
Ionian ♭2, ♭6	The (Ionic) Double Harmonic Major
Ionian ♭6, ♭7	The (Ionic) Melodic Major
Aeolian #7	The (Aeolic) Harmonic Minor
Aeolian #4,#7	The (Aeolic) Double Harmonic Minor
Aeolian #6,#7	The (Aeolic) Melodic Minor

Table 21. Standard Harmonic and Melodic Major/Minor Scales

It should be recalled that the natural modes in describing harmonic/melodic major and minor scales in Table 16 are the Ionian and Aeolian respectively. Therefore these scales are actually “Ionic” and “Aeolic” as stated only in parantheses since we avoid these adjectives in practice. However we can extend these definitions covering the scales constructed over the remaining Greek modes using the phrases “Doric”, “Phrygic”, “Lydic”, “Mixolydic” and “Locric” to emphasize their functions. As an example one may call Phrygian #6,#7 as “The Phrygic Melodic Minor”.

In that regard the scales Aeolian #7; #6,#7; Phrygian #7;#6#7; Locrian #7;#6,#7; and Dorian #6,#7 can be located instantly in Figure 1. One should expect similar formulas between the scales constructed over the Greek modes as in Table 17. To name a few critical ones we can mention

Dorian #7 = Aeolian #6,#7

Mixolydian #7 = Ionian

Mixolydian #6,#7 = Locrian #7 - Mode 2.

Aeolian #6,#7 – Mode 5 = Ionian ♭6, ♭7

Aeolian #4,#7 – Mode 5 = Ionian ♭2, ♭6

6. Musical Samples Displaying Melodic Progressions Along the Family Tree and Concluding Remarks

In this section we introduce 5 improvizational musical samples which display melodic progressions along the 5 branches of the family tree. It is observed that the mathematical order of the scales on any branch of the family tree should by no means be expected to conform to the nature of melodic progressions in a piece in view of the principles of chromatism and enharmonism. Therefore we construct a "melodic" family tree in Figure 2 by reordering the synthetic scales in the "mathematical" family tree in Figure 1.

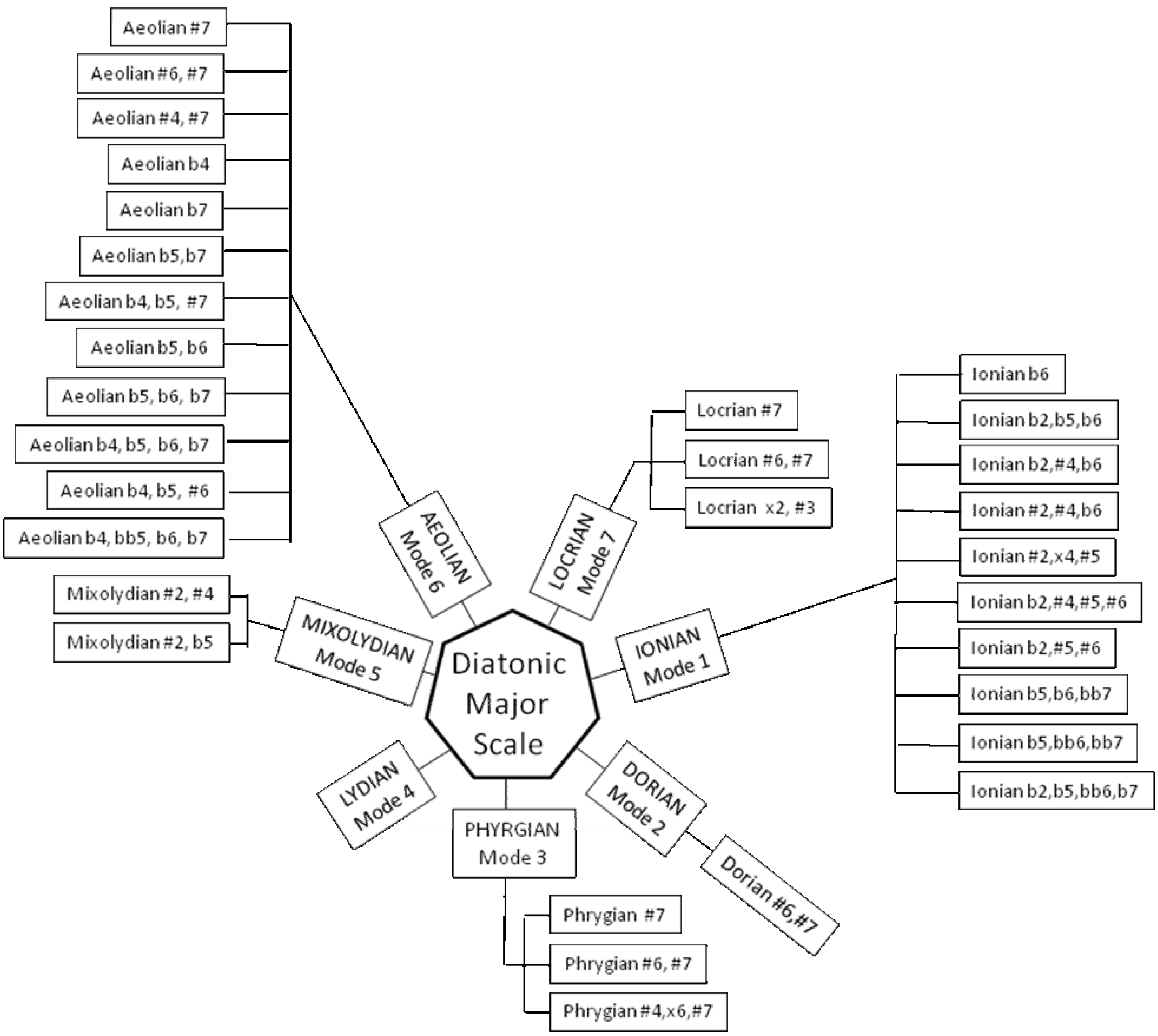


Figure 2. The melodic family tree of seven tone C scales

To demonstrate how the melodic family tree works we provide below 5 musical samples where the numbers in parantheses for each phrase stand for the order of the scales in the "melodic" family tree. Accordingly, in Example 1, (1) is the Ionian mode, (2) is Ionian ♭6, (3) is Ionian ♭2, ♭5, ♭6 and it follows until (11), the Ionian ♭2, ♭5, 𝄫6, ♭7. It applies similarly for the rest of the examples.

Example 1. A musical sample for melodic progressions over the Ionian mode


46
(1)
(2)
51
(3)
55
(4)
59
(5)
64
(6)
69
(7)
74
78
(8)
82
86
(9)
91

Example 2. A musical sample for melodic progressions over the Aeolian mode

Example 3. A musical sample for melodic progressions over the Mixolydian mode

Example 4. A musical sample for melodic progressions over the Locrian mode

Example 5. A musical sample for melodic progressions over the Phrygian mode

It is also interesting for us to observe in the examples that a number of alterations in the upper tetrachords along the branches belonging to Ionian, Phrygian and Aeolian modes, and similarly in the lower tetrachords along the branches belonging to Mixolydian and Locrian modes reveal many melodic properties specific to eastern music, which should be the subject of a forthcoming research in this area.

Through these 5 examples one can get some idea on the incorporation of synthetic scales as in Figure 2 to develop melodic progressions in a composition without ever changing the central tone in a given tonality.

When one considers the 12 notes (colors) of the chromatic scale as the tonic for any of the 224 modes we have considered, there appears as many as 224*12=2688 scales that can be incorporated in any composition under the general assumption that the harmonic progressions conform to the operational principles of the circle of fifths.

Throughout the analysis we have also observed two points which provide a good display of the parallelity relations between natural major and minor scales. The first one is in

Figure 1 where a total of 23 root scale formulas out of 32 are grouped under the natural major and minor scales. And the second is in the last two relations in Section 5 where 2 of the 12 minor root scale formulas in the family tree bring about major scales in Mode 5.

It is expected that the present theoretical investigation on the classificational aspects of the seven tone scale theory with a supplement of 5 musical examples serve as background information for the student, the educator as well as the composer.

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