Polyrhythm Analysis Using the *composite* Tool

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ABSTRACT

We introduce a computational tool that allows comparison and classification of polyrhythms in notated music. By reducing different musical textures into unpitched rhythmic strands, the *composite* tool enables visualization of the rhythmic reductions and computation of features related to polyrhythmic design, such as event density, nestedness, and polarity. The visualizations and extracted data can then be used to compare polyrhythms within a specific repertoire or between music in contrasting styles. The *composite* tool is available for online or offline use and is incorporated into the Polyrhythm Project website for exploration of polyrhythmic examples from the *Suter (1980) Corpus*.

CCS CONCEPTS

• **Applied computing → Sound and music computing**.

KEYWORDS

Music theory, Polyrhythm, Computer-aided music analysis, Corpus studies, Twentieth-century music

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1 INTRODUCTION

There has been substantial research on rhythm perception and production using simple polyrhythms, that is, two or more superposed pulse trains, each of which results in a different rate of presentation.[3][4][5][15] The findings are suggestive, but whether they extend to the complex polyrhythms found in music around the world is not clear (for a theoretical distinction between simple and complex polyrhythms, see Poudrier & Repp, 2013).[9] One challenge in conducting systematic research in the perception of ecologically valid polyrhythms is the diversity of musical practices in which they are found, such as African polyphony,[1] North Indian classical music,[2] Black Atlantic music,[12] and Metal,[7] among others. Even focusing on one relatively homogeneous musical practice, such as twentieth-century notated music from Europe and North America, the range in melodic and harmonic language, ensemble types (which vary in terms of timbre and numbers of parts), and layering techniques poses significant analytical challenges.

This paper introduces a computational tool that allows for the comparison and classification of polyrhythms in notated music. By reducing different textures to a set of single unpitched rhythmic strands, i.e., resultant or "composite" rhythms, the *composite* tool can be used to extract measures derived from the full composite and from each of the rhythmic groups that form the polyrhythm, as well as the rhythm resulting from coinciding events between groups. We begin with a presentation of the analytical applications that motivated the development and implementation of the *composite* tool, and provide details on the labeling of musical examples for use with the tool. Information on how to access the tool and documentation follows. In the final section, we discuss other applications, current limitations, and future development. Unless otherwise noted, the examples presented are from the *Suter (1980) Corpus*[11][13]; the examples are identified by the letter "R" followed by a three-digit number assigned to each example by Suter.[1]

2 ANALYTICAL APPLICATIONS

2.1 Visualization using the *composite* tool

Figure 1 presents R129, an excerpt from Leoš Janáček's *Pohádka* (Fairy Tale) for Violincello and Piano (1910; rev. 1923). The top three staves feature a digital diplomatic transcription of the notated music, while the bottom three staves present the resultant rhythms generated by the *composite* tool, with the full composite rhythm

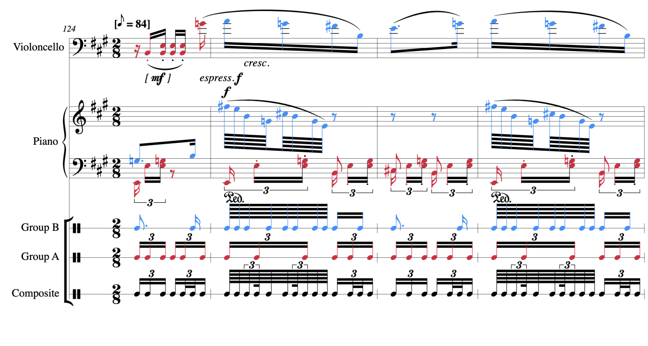

Figure 1: R129, Leoš Janáček, *Pohádka* (1910; rev. 1923), mvt. 1: Con moto, mm. 124–127. The bottom three staves are generated automatically by the *composite* tool. Grouping information is displayed in color: red for Group A and blue for Group B. [2]

[1]The *R* subset of the *Suter (1980) Corpus* includes four examples for each of the twenty composers represented in the full corpus; the *R* subset can be browsed online at https://verovio.humdrum.org/?file=poly

[2]https://verovio.humdrum.org/?file=poly/R129&filter=composite%20-gu%7Ccolorgroups

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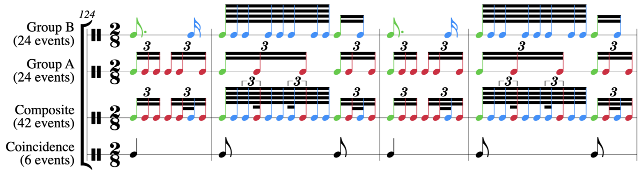

Figure 2: R129, Leoš Janáček, *Pohádka* (1910; rev. 1923), mvt. 1: Con moto, mm. 124–127. Green highlighted notes indicate coinciding events between Groups A and B; the number of events in each resultant rhythm is provided below the staff label. The regular pattern of the coincidence rhythm is strongly metric.[3]

in the lowest staff. The texture is divided into two rhythmically contrasting groups, Group A (in red) and Group B (in blue). Group attribution was done manually, with the lowest staff in the score placed in Group A by convention. Figure 2 presents an alternative visualization of the rhythmic analysis that includes the rhythm resulting from the coinciding events between Groups A and B (in green), with the interaction of the two rhythmic groups in the full composite made explicit by the coloring. The number of events in each rhythm is provided in parentheses below the staff label on the left side. In this example, the texture features duple and triple subdivisions of the eighth-note time unit, with the cello and piano right-hand consisting mainly of duple subdivisions (represented by sixteenth and sixty-fourth notes), and the piano left-hand performing triple subdivisions (triplet sixteenth notes). Despite the rhythmic conflict between these parts, the coincidence rhythm is strongly metric, with a regular long-short-short pattern in a 2:1 ratio.

By contrast, Figure 3 presents a polyrhythmic analysis of R215, an excerpt from Arnold Schoenberg's opera *Moses und Aaron* (1932) in which the coincidence rhythm suggests initial coordination of the two rhythmic groups at a slower structural level. In this example, Group A consists mainly of rhythms that align with the duple meter, with occasionally uneven subdivisions of the notated beat (mm. 81–82, beat 2). On the other hand, Group B features a dotted-quarter pulse that is out-of-phase with the half-note beat. The rhythmic pattern of Group B is also characterized by uneven time units in a 3:2 duration ratio (alternating dotted-quarter and quarter notes).

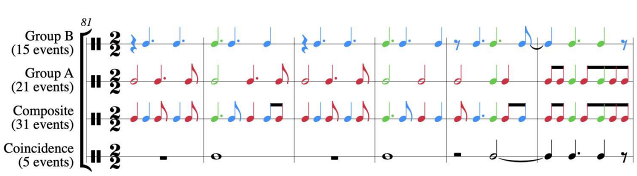

Figure 3: R215, Arnold Schoenberg, *Moses und Aaron* (1932), mm. 523–528 (Act 1, Scene 4, mm. 81–86). The low number of coinciding events and lack of coordination with the notated measure suggest a perceptibly more complex polyrhythm.[4]

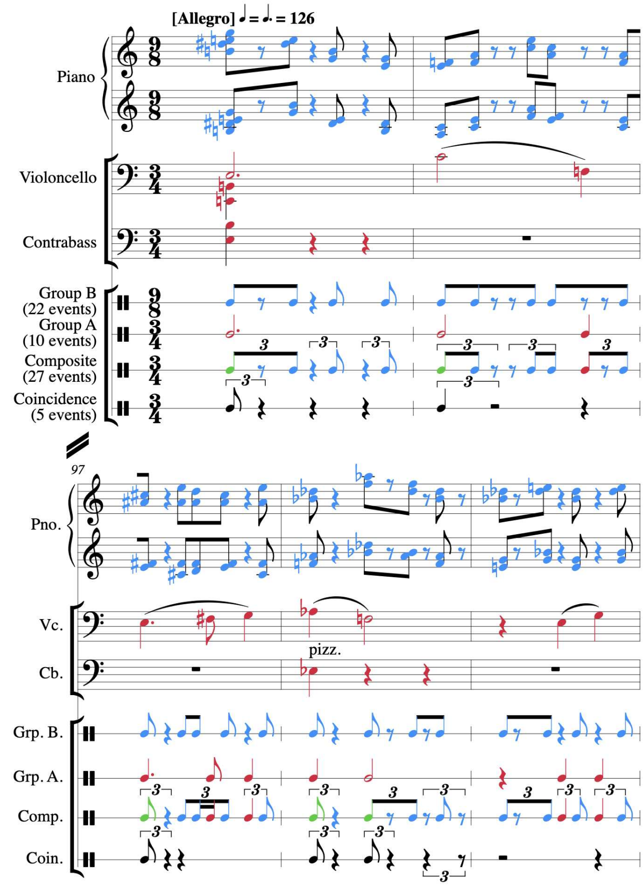

Figure 4: R599, Arthur Honegger, Symphony No. 4 "*Deliciae basiliensis*" (Pleasures of Basel) for Chamber Orchestra (1946), mvt. 3: Allegro, mm. 96–100. The polydurational groups are reduced to a single time signature of 3/4, allowing for closer examination of the interaction of Group A (in red), Group B (in blue), and the coinciding events (in green). [5]

The lower number of coinciding events and the irregular resultant rhythms that characterize each of the component rhythms result in a complex polyrhythmic structure, at least from the perspective of beat percepts.

A number of examples in the Suter Corpus are polymetric in addition to featuring polyrhythmic structures (6/80 in the *R* subset). Figure 4 presents a excerpt from Arthur Honegger's Symphony No. 4 "Deliciae basiliensis" (Pleasures of Basel) for Chamber Orchestra (1946) in which the woodwinds and the piano parts are notated with a time signature of 9/8, while the strings are notated in 3/4, with the dotted-quarter beat representing the same duration as the quarter beat, an example of *polydurational* notation. One encoding challenge with polydurational notation is how to represent a rhythm that results from a combination of two rhythmic layers that use different graphic representations for an equivalent time

[3] https://verovio.humdrum.org/?file=poly/R129&filter=composite%20-gxceumC%7Ccolorgroups

[4] https://verovio.humdrum.org/?file=poly/R215&filter=composite%20-gxceumC%7Ccolorgroups

[5] https://verovio.humdrum.org/?file=poly/R599&filter=extract%20-k1,2,6,7%7Ccomposite%20-gceuC%7Ccolorgroups

unit. The *composite* tool resolves this problem by translating the two graphic representations into one that can accommodate both rhythmic groups. In this example, the rhythmic analysis presents Groups A and B in their original graphic representations, while the composite rhythm is notated in 3/4, with dotted-quarter note subdivisions translated into triplet eighth notes. The colored composite and coincidence rhythms provide visual aids for how the different parts interact. While most of the rhythmic activity originates in Group B (in blue), Group A events (in red) provide occasional reminders of the polymetric structuring. As shown by the coinciding events (in green), this example is representative of a polyrhythm that is primarily the result of conflicting subdivisions of the measure. However, with a notated tempo of 126 quarter/dotted-quarter notes per minute, the coinciding downbeats (which occur at a rate of 42 beats per minute) are unlikely to be perceived as a guiding pulse.

A somewhat larger proportion of examples (16/80) feature changing time signatures. In these cases, the *composite* tool provides the means to observe the interaction of rhythmic design against changes of time signature. Figure 5 presents the rhythmic analysis of an excerpt from R392, Igor Stravinsky's *Perséphone* (1934). The example features a semi-regular alternation of 3/4 and 2/4 bars. The composite visualization reveals a polyrhythmic design in which Group B presents a steady stream of quarter notes that is out-of-phase with the notated bar, but fully integrated. In contrast, the coincidence rhythm features a repeating pattern of short-short-long rhythms followed by two equal longer durations (in eighth notes: 2+2+4, 2+2+4, 4, 4). Whether the coincidence pattern would result in a sense of regularity amidst a mosaic of timbres and surface rhythms is not clear, but is worth investigating further.

As demonstrated by these examples, visualization of the rhythmic analysis generated by the *composite* tool allows comparison of a wide range of polyrhythmic structures. By comparing aspects of rhythmic design within and across composite and coincidence rhythms, classification of polyrhythms from a variety of musical works becomes possible. The interaction of component rhythms and the design of the coincidence rhythm can also be suggestive of how different polyrhythms might be experienced by listeners. For example, R129 might be perceived as more metrically grounded than R215. On the other hand, the change of pace in the coincidence rhythm of R215 may result in perceived changes in the flow of the music. And while the regular coinciding events in R599 provide a unifying underlying structure, the repeating pattern of coinciding events against the changing time signatures in R392 may result in more perceptible regularity. In sum, these observations can be used to guide experimental design and contribute to the modeling of polyrhythmic experience.

2.2 Group attribution

To obtain visual representation of the full composite rhythm on Verovio Humdrum Viewer, users simply need to type the filter name "`composite`" into the filter toolbar, and press enter to activate the filter. The visualizations shown in section 2.1 involved the division of parts into two contrasting rhythmic groups (Group A and Group B). If the digital score is comprise of two parts, each notated on a separate staff, adding the option "`-g`" will automatically

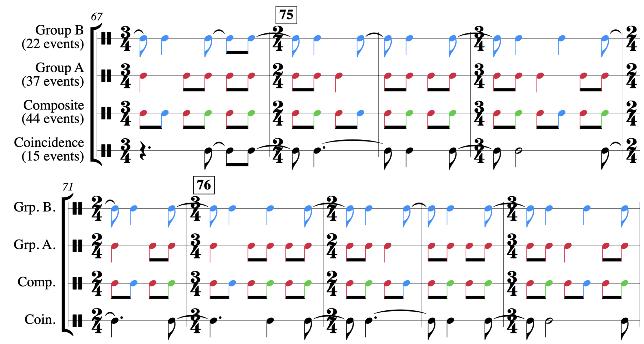

Figure 5: R392, Igor Stravinsky, *Perséphone*, Melodrama in Three Scenes for Solo Tenor, Choir and Orchestra (1934), II. Perséphone aux enfers (Persephone in Hell), mm. 67-75. The coincidence rhythm reveals a regular pattern that runs against the changing time signatures. [6]

assign the lowest staff to Group A (`*grp:A`) and the upper staff to Group B (`*grp:B`). If an example features more than two staves, group attribution can be done semi-automatically by using the filter "`composite -G`" followed by the desired staves grouping. For example, given a score written for string quartet, the expression "`-G 1,3:2,4`" will render a grouping with the cello (staff 1) and violin II (staff 3) in Group A and the viola (staff 2) and violin I (staff 4) in Group B. If working with a corpus of examples with the same configuration of parts or staves, group attribution can be batch-processed on the command line.

The implementation of the composite tool was motivated by the Polyrhythm Project, which aims to model polyrhythmic experience. To that end, we selected a corpus of musical examples featuring superposed rhythms built from different subdivisions or groupings of metric units. The corpus is comprised of examples written for a wide range of ensemble types, from solo keyboard works to full orchestra, with and without vocal parts. Because of the varying number of parts and complexity of their rhythmic structure, these examples required manual group attribution. To establish ground truth, the task of dividing the full texture of sounding parts into two contrasting rhythmic groups was assigned to three graduate music theory students. The students were provided with 67 unidentified musical examples comprised of 3 to 30 parts, with staves numbered from the lowest to the highest. (The 12 examples that featured only two staves were not assigned as each staff could be designated as a group by default; one example featured only one staff and was later re-transcribed with the full orchestra.) Students were instructed to "separate the parts into two groups based on how similar they are to each other from the perspective of rhythmic structure." Rhythmic groups were identified as "comprised of the parts that are most similar to each other in terms of rhythmic structure, with parts being most dissimilar across groups."

Initial group attribution was fairly consistent across the three students, with only eight examples requiring further examination by two project team members (including the principal investigator).

[6] https://verovio.humdrum.org/?file=poly/R392&filter=composite%20-gcexC%7Ccolorgroups

For these examples, group attribution was determined by majority rule. To obtain a more fine grain separation that takes into account changes of rhythmic structure within parts, visual examination of each score and group attribution at the level of the notated bar were performed by the principal investigator. A pre-established set of criteria was used, which included grouping ambiguous bars with the parts that featured the greatest number of coinciding events. In cases where the number of coinciding events was the same across rhythmic groups, group attribution favored grouping consistency.

The final group attribution is encoded in the Polyrhythm Project Sampler available on Verovio Humdrum Viewer[7] as well as on GitHub.[8]

2.3 Composite measures

One feature of polyrhythmic structures (as compared to rhythmic structures that use subdivisions that are consistent with a given time signature) is an increased number of events within a given time unit (beat, measure, or second). For example, while the cello part in Figure 1 presents two to five events per bar and the piano part presents six (left hand) to eight (right hand) events per bar, the full composite rhythm presents seven to 14 events per bar. As demonstrated in the section above, the number of coinciding events also provides some insight into the interaction of the two contrasting rhythmic groups and is suggestive of the complexity of the polyrhythmic design. Taking these features as a starting point, four computational measures were derived from the rhythmic data generated by the *composite* tool: composite event density, event density ratio, nested ratio, and polarity. These features can be automatically calculated by the *composite* tool.[9]

Event density can be calculated in several different ways, depending on the chosen referential time unit. Here, we calculate event density in units of seconds from the full composite rhythm based on performance times extracted from randomly selected recordings in the Naxos streaming library:

$$\text{composite event density} = \frac{\text{composite events}}{\text{performance time [sec.]}} \quad (1)$$

Event density can also be computed as a ratio between group densities, providing a measure of the relative influence of each group on rhythmic activity. Here is the equation for measuring relative densities between two separate rhythmic groups:

$$\text{event density ratio} = \frac{\text{Min(group A density, group B density)}}{\text{Max(group A density, group B density)}} \quad (2)$$

A value of 1 indicates both groups have the same density, while values approaching 0 indicate a more unbalanced relation between the two groups. As shown in Table 1, while R129 (Fig. 1) and R389 (Figure 12, below) present similar composite event density measures, they contrast in terms of event density ratio, which is suggestive of a more unbalanced rhythmic design in R389.

Nestedness, that is, the property by which an event belongs to one or more rhythmic strands,[8] is another aspect of polyrhythmic structuring that can contribute to the relative independence of superposed contrasting rhythms, and by extension, the perceived complexity of a given polyrhythm. The nested ratio is calculated based on the number of coinciding events in the full composite:

$$\text{nested ratio} = \frac{\text{coincidence events}}{\text{composite events}} \quad (3)$$

Lower nested ratios indicate more independent rhythmic strands, while higher values suggest stronger integration, which may result in a weaker perception of contrasting rhythmic streams. For example, if considered in isolation of other features, the lower nested ratios of R129 (Fig. 2), R215 (Fig. 3), and R599 (Fig. 4) indicate that the two rhythmic streams in these polyrhythms are fairly independent, while the relatively higher nested ratios of R279 (Figure 10, below) and R389 (Fig. 12) suggest that the two rhythmic streams are somewhat more integrated.

The number of events in each group can also be used to quantify the *polarity*, that is, the relative salience of rhythmic groups.[8] As measured here, polarity can serve to identify the dominant rhythmic group in a given polyrhythm:

$$\text{polarity} = \frac{\text{group A events} - \text{group B events}}{\text{composite events}} \quad (4)$$

Because it compares the number of events between groups, polarity is related to event density ratio, with 0 (instead of 1) resulting from an equivalent number of events in the two rhythmic groups. Polarity can also be calculated as a ratio by using the absolute difference between Groups A and B events. Higher absolute polarity indicates a less balanced rhythmic design, which may be conducive to higher salience of one group over the other. As measured here, positive polarity indicates Group A dominance, while negative polarity indicates Group B dominance. In Table 1, the polarity measures for R279 (Fig. 10) and R599 (Fig. 4) indicate Group B dominance, while the remaining polarity measures indicate Group A dominance, except for R129, where the two groups are equally salient in terms of number of events.

Table 1: Polyrhythmic measures for six sample corpus examples.

ID	Density	Density Ratio	Nested Ratio	Polarity
R129	8.4	1	0.143	0
R215	1.722	0.714	0.161	0.194
R279	1.714	0.5	0.5	-0.5
R389	6	0.542	0.542	0.458
R392	1.967	0.6	0.317	0.333
R599	4	0.5	0.194	-0.403

While these measures provide a set of objective observations for the classification of polyrhythms, they can also be used to test the influence of polyrhythmic features on listeners' perceived musical qualities. In an exploratory listening experiment with 72 examples from the *R* subset,[10] it was found that composite event density was positively correlated with perceived mood, energy, and desire to move along with the music, as well as higher ratings of the qualities “Exciting”, “Structured”, and “Complex”. Nested ratio was also positively correlated with perceived mood, desire to move along with the music, “Exciting”, and “Structured”. On the other hand, event density ratio and polarity ratio were found to be negatively correlated with a number of these perceived qualities, although the

[7] https://verovio.humdrum.org?file=poly
[8] https://github.com/polyrhythm-project/rds-scores
[9] See reference documentation for the tool at https://doc.verovio.humdrum.org/filter/composite

effect was generally weaker, except for the negative correlation between polarity ratio and "Complex". These preliminary findings attest to the perceptual relevance of composite measures in an ecological context.

2.4 Composite rhythm variability

Another useful measure to apply to composite rhythms is nPVI (normalized pairwise variability index), which can be used as a general measure of rhythmic complexity.[6] It is defined by the equation:

$$\text{nPVI} = \frac{200}{N-1} \sum_{n=1}^{N-1} \left| \frac{x_n - x_{n+1}}{x_n + x_{n+1}} \right| \quad (5)$$

where x is a list of composite rhythms of length N.

Lower nPVI scores for a composite rhythm indicate more repeated rhythmic values, and higher scores indicate more note-to-note changes in the composite rhythm durations. These scores can then be used to group pieces for analysis or searching by similar composite rhythm variability.

An example application of the nPVI metric is illustrated with an analysis of the 48 J.S. Bach Well-Tempered Clavier fugues.[10] Table 2 lists the nPVI for each fugue, from lowest to highest. Low nPVI values indicate long regions of repeated notes, while high nPVI values usually indicate repeating cycles of rhythms and shorter repeated note patterns.

Table 2: Bach WTC fugues, sorted from lowest to highest nPVI score for composite rhythms.

Fugue	nPVI	Fugue	nPVI	Fugue	nPVI
wtc2f04	0.20	*wtc2f23*	3.75	*wtc2f03*	11.07
wtc1f10	0.50	*wtc1f03*	4.17	*wtc2f13*	11.42
wtc2f21	0.72	*wtc1f13*	4.39	*wtc1f23*	11.93
wtc1f09	0.91	*wtc2f10*	4.40	*wtc1f20*	12.41
wtc2f12	0.92	*wtc1f14*	4.45	*wtc1f02*	14.73
wtc2f19	1.23	*wtc2f24*	4.70	*wtc1f22*	15.92
wtc1f17	1.40	*wtc2f18*	4.80	*wtc1f01*	17.07
wtc2f15	1.71	*wtc1f21*	5.59	*wtc2f09*	17.17
wtc2f01	1.84	*wtc1f11*	6.04	*wtc1f12*	18.07
wtc2f11	2.47	*wtc1f06*	6.63	*wtc2f22*	19.33
wtc1f24	2.59	*wtc2f02*	7.06	*wtc1f18*	19.41
wtc2f05	2.64	*wtc2f16*	7.39	*wtc2f08*	21.18
wtc1f07	2.72	*wtc1f04*	7.65	*wtc2f07*	23.65
wtc1f19	2.93	*wtc1f15*	8.27	*wtc1f05*	25.18
wtc2f06	3.10	*wtc1f08*	8.39	*wtc1f16*	29.53
wtc2f17	3.54	*wtc2f14*	8.88	*wtc2f20*	31.67

Figure 6 gives samples of the initial composite rhythm for the lowest and highest nPVI scores. Notice that the top three staves from *wtc2f04* all have the same rhythm (throughout the entire score). By contrast, the bottom three lines of composite rhythm are from *wtc2f20*, where the beat is usually broken up into a pattern of long and short rhythms throughout the fugue.

[10] Explore composite rhythms of the fugues starting from the url https://verovio.humdrum.org/?file=bach-wtc-fugues&filter=composite%20-ul70

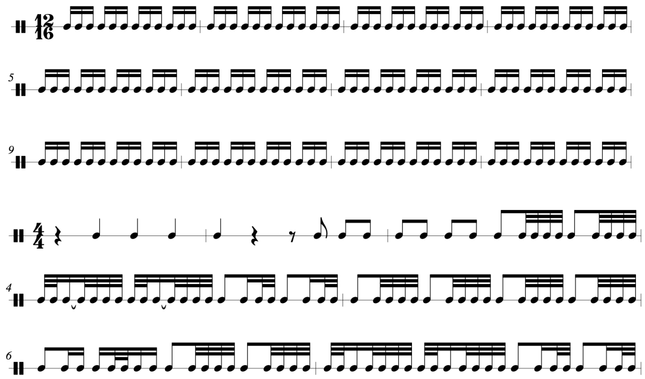

Figure 6: Opening measures of fugues with lowest (wtc2f04, first three systems) and highest (wtc2f20) nPVI scores in Table 2. [11]

The composite rhythms of the *R* subset from the Suter Corpus have an average nPVI value of 30, which is just slightly less than that of the Bach fugues' highest nPVI value. This is to be expected, since Groups A and B present contrasting rhythmic subdivisions that will tend to generate a more variable full composite rhythm. In the exploratory experiment reported above,[10] nPVI group difference, which corresponds to the difference between the two rhythmic groups' nPVI values, was found to predict higher ratings of perceived energy, "Exciting", and "Complex". These preliminary results are consistent with the use of nPVI as a measure of complexity, and provide additional insight on perceived qualities related with higher nPVI values.

3 ACCESSING THE *COMPOSITE* TOOL

The *composite* tool is available as a low-level batch-processing program for unix-like terminals as well as several web-based user interfaces: The Polyrhythm Project website is an easy-to-use interface for exploring the Suter Corpus that includes buttons to toggle various options of the *composite* tool. Verovio Humdrum Viewer (VHV) is an online digital score editor that includes several music processing and analysis tools provided by *humlib* that can be applied to built-in repertories such as the Suter Corpus[12] as well as user-supplied scores, primarily in the Humdrum format, but also through translations from the MusicXML, MEI and Musedata formats. The JavaScript-based Humdrum Notation Plugin interface[13] to Verovio allows you to create your own webpages containing digital scores that can then use the *composite* tool to display composite rhythms and extract numeric analyses.

3.1 C++ source code and command-line usage

Source code for the *composite* tool is available in the humlib repository on Github.[14] Humlib is a C++ code library for parsing music

[11] https://verovio.humdrum.org/?file=bach-wtc-fugues/wtc2f04.krn&filter=composite and https://verovio.humdrum.org/?file=bach-wtc-fugues/wtc2f20.krn&filter=composite
[12] https://verovio.humdrum.org/?file=poly
[13] https://plugin.humdrum.org
[14] https://github.com/craigsapp/humlib

Table 3: Essential options for *composite* tool analysis.

Option	Description
none	Display full composite analysis.
`-g`	Display group composite analyses.
`-c`	Display note-attack coincidences between groups.
`-F`	Do not display full composite analysis.
`-C`	Color notes in full composite rhythm by group.
`-e`	Show event counts before first system.
`-x`	Show only analysis staves and not input music.

in the Humdrum digital score format[15] and includes a command-line interface to the *composite* tool. Humlib is also integrated into Verovio,[16] a music-notation rendering library written in C++ that provides interfaces for the command-line, JavaScript and Python.

All interfaces interact with the *composite* tool using unix-style command-line options, with a full list available on the VHV documentation website.[17] Table 3 lists the main options for displaying composite rhythm analyses generated by the *composite* tool.

The Polyrhythm Project website hides the technical implementation details of these options, while Verovio Humdrum Viewer uses a similar interface as that of the command-line version of the tool.

3.2 Verovio Humdrum Viewer

The *composite* tool is available in Verovio Humdrum Viewer,[18] both for built-in digital score repertories as well as user-encoded Humdrum scores and online conversions from MusicXML, MEI or Musedata formats. The Suter Corpus can be browsed on VHV using the display style of Figure 1 by using the filter seen in the filter toolbar at the top right corner of the webpage in Figure 7.[19]

Figure 7 demonstrates turning on most visual display options in the *composite* tool, such as making the input score smaller and colorizing the different groups and coincidences. The filter shown in Fig. 7 is "`composite -gu | colorgroups`", where the "`-g`" option adds the group composite rhythms, and the "`-u`" option displays the composite rhythm staff notes with up-stems. Then the *colorgroups* tool is applied to display each group with a different color.[20] Clicking on the left/right-arrow buttons at the top left corner of the website allows browsing through the repertory, or you can click on the up-arrow button to view an index of all examples.

Once such a display is generated in VHV, it can be saved in SVG or PDF formats from the *File* menu.[21]

[15]https://www.humdrum.org
[16]https://www.verovio.org
[17]https://doc.verovio.humdrum.org/filter/composite
[18]https://verovio.humdrum.org
[19]The filter can also be embedded directly in a URL, such as http://verovio.humdrum.org/?file=poly/R129&filter=composite%20-gu%7Ccolorgroups
[20]See https://doc.verovio.humdrum.org/filter for more details about using the filter toolbar in VHV.
[21]Music notation examples for this paper were generated using VHV and downloaded as SVG images, for example.
[22]https://verovio.humdrum.org/?file=poly/R129&filter=composite%20-Ccmgul79%7Ccolorgroups

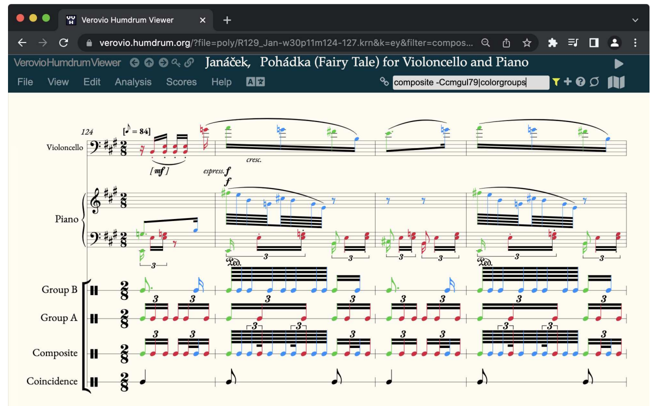

Figure 7: Example use of the *composite* tool in Verovio Humdrum Viewer.[22]

3.3 Polyrhythm Project website

The Polyrhythm Project website[23] provides a simple user interface for the *composite* tool for working with the Suter Corpus. Work and single-example pages on the website display a row of buttons above musical examples that allow selecting a variety of composite rhythm displays as demonstrated in Figure 8.

Table 4: Meaning of the *composite* tool options used in Fig. 7.

`-g`	Show group composite rhythm staves.
`-c`	Show composite rhythm staff.
`-l 79`	Shrink input score to 79% of original size.
`-m`	Mark group coincidences in score (in green).
`-C`	Color groups/coincidences in full composite rhythm.
`-u`	display composite rhythm staves notes with up-stems.

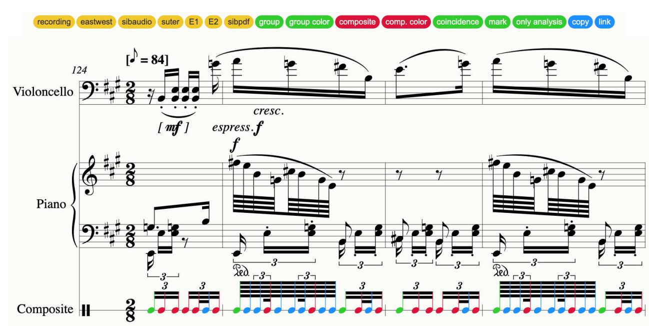

Figure 8: Polyrhythm Project example page displaying *composite* tool analyses. Yellow buttons show available source materials (audio and graphic). Green buttons give composite rhythm analysis options, with the red buttons for "composite" and "comp. color" already selected to show the composite rhythm with group colorings. Blue buttons are used to copy the music data and the page link. [24]

[23]https://polyrhythm.humdrum.org
[24]https://polyrhythm.humdrum.org/example/?id=129

The Polyrhythm Project website also includes metadata on the composition, premiere, and publication of the works from which the examples were extracted. Specific measure numbers of examples within the full score and information about reference recordings for each example are provided, with start and end timestamps. The website also features several maps that provide an overview of the corpus, including an interactive timeline map of the premiere of each work. It also includes additional information about the project, editorial policies, and other documentation.

4 CONCLUSIONS

4.1 Other applications

Beyond the Polyrhythm Project, the *composite* tool has many other possible applications for rhythm analysis. It could be used for cross-genre and cross-cultural comparative research on the rhythmic aspects of music, including formal design. As a starting point, Verovio Humdrum Viewer provides several collections of encoded scores, many of which are comprised of keyboard music, the composite rhythmic analysis of which can be automatically generated. Beyond composer-based analysis, genre-based comparative analysis could be performed (for example, comparing Viennese sonatas by Franz Joseph Haydn, Wolfgang Amadeus Mozart, and Ludwig van Beethoven). Other composers featured include Domenico Scarlatti, Fryderyk Chopin, and Scott Joplin. In addition to the preludes and fugues from the Well-Tempered Clavier, the platform also includes Johann Sebastian Bach's Chorales, composed for four vocal parts that can also be displayed on two staves. The automatic generation of the full composite and its component rhythms makes it easy for users with different levels of computational skills, which could facilitate use of the tool in an educational context.

The *composite* tool could also be applied to several different types of activities in the domain of the performing arts and music composition. Conductors and performers can use it to study the rhythmic interaction of parts in a given work, which could prove to be especially helpful for complex polyrhythmic textures, which can be challenging in terms of coordination of the ensemble parts. Using the *composite* tool in combination with the "`extract -s`" filter, conductors and performers can also select specific parts of an ensemble they wish to examine more closely. For example, a performer may wish to study how their part rhythmical integrates with other parts in an ensemble. In addition, composers can use the tool to study polyrhythmic structuring in their own and others' music. The colors used for visualizing events divided by groups as well as coinciding events can facilitate the elaboration of different types of textures. The addition of "`-o`" and a specific group designation ("A" or "B") to the composite filter will further allow users to hear the pitches in a given component rhythmic group, as demonstrated in Figure 9.

For computational analysis, the tool can be used to automatically group parts by rhythmic similarity. It can also be used to rank search results by rhythmic complexity. With a growing number of

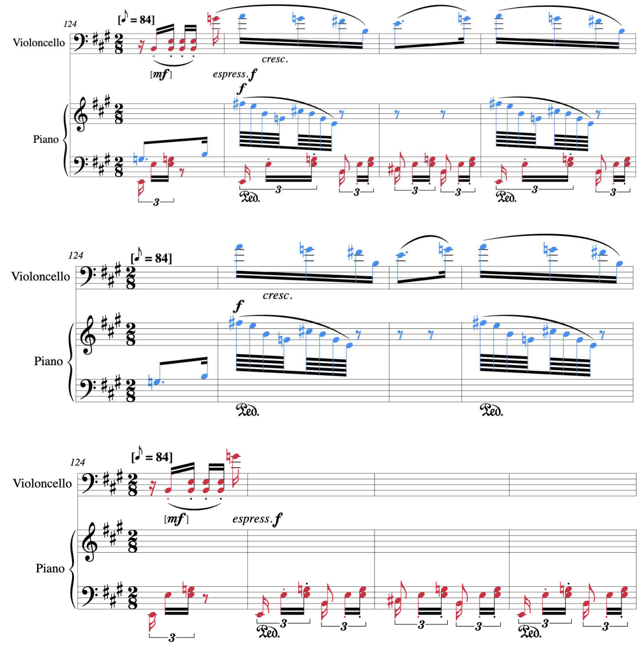

Figure 9: R129, Leoš Janáček, *Pohádka* (1910; rev. 1923), mvt. 1: Con moto, mm. 124–127. Extracting only notes from Group A (red) or Group B (blue) by using the "`-o`" option allows visualization and audition of individual component rhythmic groups. [25]

online digitized scores, digital librarians and MIR researchers may find the tool useful for classification based on rhythmic features.

4.2 Limitations and future development

Currently only two polyrhythmic composite streams can be processed by the *composite* tool. More could be implemented, although the perceptual relevance of adding more is unclear for its current purposes. More importantly, although the interaction of rhythmic strands is an important aspect of polyphonic music, the experience of music listening involves many more musical features, including timbre, melody, and harmony. Even from the perspective of polyrhythmic design, contrasting rhythmic groups may be the result of other features such as melodic or accentual patterning. One important aspect of future work will be to focus on how accentual and pitch-based features interact and modulate the experience of polyrhythm. For example, the composite rhythm analysis of R279, shown as Fig. 10, presents a simple 2:1 relationship between groups, with Group A being fully integrated (in green). However, a closer look at the divided texture on the score (Figure 11) reveals a 3:2 polyrhythm that results from the melodic grouping of a slurred pattern of four eighth notes in Group B (in blue) against the 3/4 meter supported by accented events every three quarter notes in Group A (in red).

[25] Links to each sub-example on VHV: https://verovio.humdrum.org/?file=poly/R129&filter=colorgroups, https://verovio.humdrum.org/?file=poly/R129&filter=composite%20-oB%7Ccolorgroups, and https://verovio.humdrum.org/?file=poly/R129&filter=composite%20-oA%7Ccolorgroups.

[26] https://verovio.humdrum.org/?file=poly/R279&filter=composite%20-xgcuemC%7Ccolorgroups

[27] https://verovio.humdrum.org/?file=poly/R279&filter=extract%20-k2,4%7Ccolorgroups

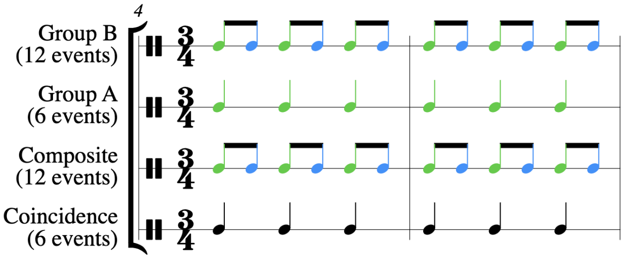

Figure 10: R279, Maurice Ravel, *Rapsodie espagnole* (*Spanish Rhapsody*) for Orchestra (1908), I: Prélude à la nuit (Prelude to Night), mm. 4–5; rhythmic analysis only. Groups A and B stand in a simple 2:1 relationship, which is not characteristic of polyrhythm. [26]

Similarly, in Figure 12, while Group B in R389 appears to be fully integrated, the notated music (Figure 13) shows a 7:4 polyrhythm resulting from a melodic pattern of seventh sixteenth notes in the violin part, with the first event of each group of seven being marked by a trill.

Since the Suter Corpus represents the first half of twentieth-century classical music by twenty composers and a wide range of ensembles, there are numerous complexities when generating dynamic notation for the scores and composite rhythms. Many notational enhancements have been added to the notation of Humdrum scores over the past few years, such as percussion instruments and cross-staff and cross-bar beams. Also, complicated polyrhythmic interactions, such as tuplets involving larger prime numbers are difficult to merge into a single composite rhythm, and usually require nested tuplet notations which are yet to be implemented.

More work will be done in the future to display polymetric music as shown in Fig. 4. The technology for displaying polymetric and polydurational notation is still in development, which limits visual analysis with the *composite* tool. Composite rhythms can

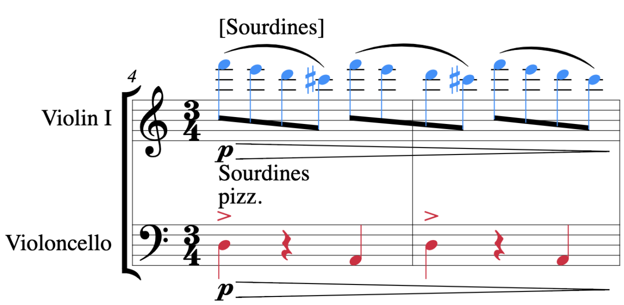

Figure 11: R279, Maurice Ravel, *Rapsodie espagnole* (*Spanish Rhapsody*) for Orchestra (1908), I: Prélude à la nuit (Prelude to Night), mm. 4–5; partial view of notated music with colored groups. Group B (in blue) features rhythmic groupings of four eighth notes that run against the notated bar, which is projected by dynamic accents every three quarter notes in Group A (in red), resulting in a 3:2 polyrhythm. [27]

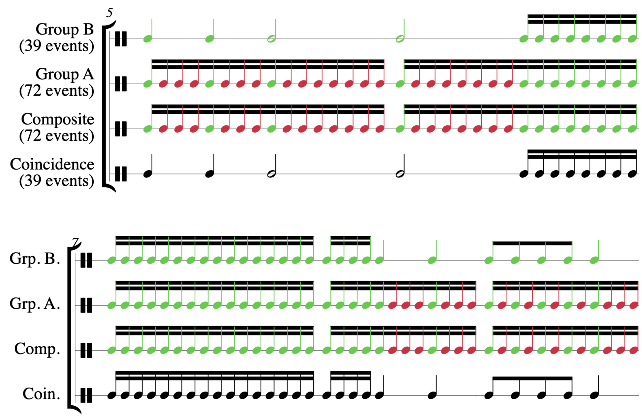

Figure 12: R389, Igor Stravinsky, Duo Concertant for Violin and Piano (1932), Églogue I (Part II), m. 5. Group B (in green) appears to be fully integrated within Group A, suggesting a simple rhythmic relationship between groups. [28]

also be the starting points of other types of music analysis. In *Harmonic Rhythm*, particularly chapters 2 and 3, Swain discusses the reduction of textural (composite) rhythm into phenomenal harmonic rhythm and interactions with the bass part's rhythm to generate harmonic rhythm in tonal music.[14] This sort of analysis is similar to polyrhythmic analyses of non-tonal and extended tonal music where multiple streams of rhythms interact in the composition and perception of the music.

Figure 13: R389, Igor Stravinsky, Duo Concertant for Violin and Piano (1932), Églogue I (Part II), m. 5. The presence of a trill every seven sixteenth notes and the melodic contour in the violin result in a 7:4 polyrhythm between parts. [29]

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[28]https://verovio.humdrum.org/?file=poly/R389&filter=composite%20-gxucem%7Ccolorgroups

[29]https://verovio.humdrum.org/?file=poly/R389&filter=composite%20-g%7Ccolorgroups

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