“Visualizing Macrocompositional Dynamics in the Work of Iannis Xenakis”

Presented at CIM08, Thessaloniki, 6 July 2008

Charles Turner, Graduate Center, City University of New York: <cturner@gradcenter.cuny.edu>

Marcel Hendrix, Eindhoven Technical University

(Partially revised as read; figures at bottom.)

In addition to the conference organizing committee and my collaborator, Marcel Hendrix, I'd like to thank Agostino Di Scipio and Jim Harley for their small, but critical assistance during the preparation of this presentation.

The impetus for my talk today stems from a presentation I made in Toronto at the Xenakis Legacies Symposium in 2006. In addition to some research into the historical context of Xenakis' use of cellular automata, work pioneered by Peter Hoffmann and Makis Solomos, I made some brief observations on the character of Xenakis' “composing robots,” the various automata or algorithms that Xenakis created and used throughout his life as a composer. My curiosity about the large-scale organization and rhythms of his works were further piqued by a 2004 paper by the Musical Representation Group at IRCAM, which argued for a “logical time domain” within which Xenakis determined the succession of the sound complexes of a particular piece.

The analysis of Xenakis' music poses special problems. Joseph Kerman, in his book *Contemplating Music*, makes the point that the history of modernist music theory is inextricably bound up with the problems of creating music, rather than elucidating it. {p. 106} This general problem of twentieth century analysis is further complicated by Xenakis' rejection of serialism, in many ways the central thread of post-tonal theory. Xenakis' interest in massed sounds, and the resulting harmonic implications, created a unique creative legacy. As a result, much of the analytical work done on Xenakis' has focussed on the early compositions so well documented in *Musiques formelles*. Many analytical essays have had to deal with the formidable task of simply describing, or accounting for, the specific compositional methods he utilized.

But Xenakis, like many twentieth century composers, believed that new musical theories could be constructed. These theories would provide a new system, or a kind of universality, in which an individual work could attain its validity. Although far from the immediate concerns of the Common Practice period, these modernists still retained, in Robert Morgan's words, the “dialectic between work and system.” As Morgan suggests of the epoch of tonality: “It was the responsibility of theory and analysis, then, to reveal, on the one hand, the extent and individuality of these departures, and on the other, to show how, on some level, the agreed upon assumptions of musical order and logic remained nevertheless unimpaired.” {p. 38} The global network of associations that constituted the Post-War musical avant-garde is a testament not only to modern means of communication, but a shared search for system by these composers. It was a search for the new, but also a desire to preserve a connection to the fine art music system established over several centuries.

My contribution today is a particular analytical view of Xenakis' work. I will attempt a partial analysis of the “logical time domain” of several of his works. As you will see, this presentation is not so much focussed on results, as it is on suggesting a way to proceed with the analysis of Xenakis' compositions. I will look at four compositions for string orchestra: *Syrmos* of 1959, *Aroura* of 1971, *Shaar* of 1983, and *Voile* of 1995. This selection draws from a compositional genre Xenakis was well know for: strings, and these four compositions constitute practically all his writing for this instrumental configuration. In addition, the dates of composition fall roughly into each decade of Xenakis' musical career. *Syrmos* is known to be composed using the same methods as *Analogique A & B* of 1958, and so is well-documented in *Musiques formelles*. These compositions are approached not to explain Xenakis' theories, or how sketches resulted in a particular score, but to demonstrate an analysis that treats the scores in a unified manner. Further, the analytical method attempts to recognize Xenakis' compositional theories without however, being identical with them. In its focus on the score, the analytical method attempts to address the music "as heard," as well as composed.

In *Musiques Formelles*, Xenakis states that *Syrmos* was composed using the same mathematical structure as *Analogique A & B*. {p. 81} Two chapters of *Musiques formelles* are devoted to the theory of *Analogique*, and its application to the two compositions: one for strings and the other for electronic tape. (SEE FIGURE 2) *Analogique A* is composed from a succession of "screens," a three dimensional matrix of pitch, intensity and event density groupings. These screens, which have a theoretical connection to Dennis Gabor's representation of sound as acoustic quanta, succeed each other at a rate of 1.2 per second. A screen's pitch/intensity/density matrix determined Xenakis' calculation of the sound complex for that 1.2 second period. (SEE FIGURE 3) For *Analogique A*, Xenakis worked with a set of eight screens having different pitch, intensity and density distributions. A particular succession of screens, and their composition into sound complexes, constitute the composition.

(SEE FIGURE 4) The succession of screens in *Analogique A* is determined by a finite automaton that Xenakis created, which has the further property of not remembering its previous states of transition. The transition to a future state is decided by the specific probability of its occurrence. This automaton, or properly Markov Chain, exists in an intermediate level between the "outside time" domain of Xenakis' screens, and the "inside time" domain of the composed sound complexes. Xenakis' creation and manipulation of this automaton created the large-scale temporal organization of *Analogique A*, as well its rhythmic hierarchy.

Ideally, a temporal analysis of *Syrmos* could make use of the published descriptions of *Analogique A*. (SEE FIGURE 5) However, only a brief excerpt of the screen successions are published for *Syrmos*. In addition, it seems not possible to work backwards from the score and description of *Analogique A* to derive Xenakis' working method. The specific details of composing the sound complexes are not published, and more importantly, Xenakis' published no description of how sound event densities at the screen level are translated into the massed glissandi that are so much a part of *Syrmos*, but not *Analogique A*.

(SEE FIGURE 6) A solution could, however, be to simply catalog the pitches, dynamics markings, and quantity of notes found in the score for each screen interval in *Syrmos*. One could establish a set of screens, with their respective groupings in a pitch/intensity/density matrix. From the succession of screens, one could then recover a Markov Chain that would replicate the probabilities of transition found in *Syrmos'* score. This automaton might not be the one that Xenakis used to compose *Syrmos*, but it would be one capable of composing another piece musically equivalent to it.

As my goal was to examine a selection of compositions, not just one, the task of developing note databases for these orchestral works proved overwhelming. The analysis of excerpts was unsatisfactory, because they do not contain enough of the overall character of the composition. The solution was to catalog a more general property of the entire score as revealing, or suggestive, of the anticipated detail a full note database would reveal. This database, in the case of *Syrmos*, consists of “silence” by instrumental voice at the same level of time granularity as one of *Syrmos'* screens. The eighteen strings of *Syrmos* are considered as eighteen partially overlapping pitch groups. With *Syrmos'* screens organized on the half-measure, and its tempo at 56-58 half-notes per minute, silences of .95 seconds represent areas of zero sound event density, and also zero intensity. (SEE FIGURE 7) The resulting database, with silences shown in white, is surprisingly informative about the overall temporal quality of the composition, showing both the extents of the resulting sound complexes within the overall composition, and also something of the character of the complexes themselves. In many cases, the staggered entry of massed glissandi are visible, as are the areas of horizontal parallel instrumental lines.

The other three compositions were treated in the same manner, although different time granularities were chosen. *Aroura's* tempo of sixty half-notes per minute suggested keeping the same half-measure time slice, but *Shaar's* tempo of forty-six quarter-notes, and *Voile's* tempo of forty-five eighth-notes suggested a diminution to those values to keep all time slices in the range of a one-second granularity. It should be noted that there is no published indication that Xenakis continued to use the screen concept of *Analogique* and *Syrmos* in these later compositions. *Aroura* uses the same rhythmic grouping as these earlier pieces, where the half-measure is often divided into quarter-note triplets and quintuplets. But by the 1980s and the composition of *Shaar*, Xenakis seems to have abandoned this scheme.

(SEE FIGURE 8) Looking at the "silence maps" of these compositions, there appear striking differences between them. *Aroura* shows an organization of voices that are more often either sounding together, or silent. Transitions are more often made in unison by the voices, rather than by gradual entry. There are also lengthy solo passages, and passages with only a few voices in play. *Shaar* shows evidence of almost constant sounding of all voices, and *Voile* shows evidence of its spare, almost architectural, interplay of voices.

(SEE FIGURE 9) These silence maps can be further sorted by voice-count, and reduced to a kind of distribution graph. With silence represented in white again, *Syrmos* shows a lengthy, almost linear tapering off toward a single half-measure of total silence, this toward the end of the composition. *Aroura* shows a much more rapid reduction to just a few voices after a much greater quantity of tutti playing. *Aroura* also shows the greatest use of pure silence of the three works. *Shaar* starts to exhibit "plateaus" of voices, which by the time of *Voile's* composition, appear much more marked and longer in duration.

(SEE FIGURE 10) Beyond these impressionistic looks at the data, one can construct transition tables of the successions of screens, or time slices. The sequence of the count of silent voices from beginning to end would express the temporal character of the composition, but result in an enormous, determinate table with thousands of states and transitions. To reduce the level of detail, transitions can be enumerated as a percentage of the total voice count. For example, in *Voile*, eighteen voices silent at the eight-note level are followed twice by a repeat of eighteen silences, three times by a reduction to sixteen silences, and once by a reduction to seventeen. Therefore, we can say that eighteen voices remain silent in 33% of the succeeding time slices; are reduced to seventeen in 50% of the transitions; and to sixteen in 17% of the transitions. (SEE FIGURE 11) By resorting to a percentage accounting, we can suppress some of the complex detail, and gain a higher level portrait of the voice transitions in one of Xenakis' compositions. This percentage reduction enables the consideration of a transition table no larger than the square of the number of voices in the composition, plus one: in our consideration ranging between thirteen by thirteen for *Aroura*, and twenty-one by twenty-one for *Voile*.

This percentage accounting suppresses details by creating an indeterminate picture of the actual dynamic behavior of the voices. This statistical picture replaces determinate details with probabilities because the aggregate behavior is thought to be revealing of the composition's dynamics. Further, consideration of time slices is limited to the current slice, and the transition to the next future slice: there is no memory of the past history of transitions. Transition tables constructed in this manner are, of course, Markov Chains.

(SEE FIGURE 12) The transition tables, here informally divided into quadrants, show a difference in character between the compositions. The density of the upper left and lower right quadrants give a sense of the degree to which silent voices tend to transition to neighboring counts. In *Syrmos*, for example, the great density of the upper left quadrant suggests that when less than fifty percent of the voices are silent, they tend to remain in that state, even if the exact number varies. The other quadrants, those of the upper right and lower left indicate the density of transitions from less than half, to more than half silent voices, and vice versa. These quadrants are much less dense in *Syrmos*, and indicate the slight degree of sudden change in intensity within a half-measure's time period. By contrast with *Syrmos*, *Aroura* seems much more given to rapid changes in instrumental intensity, and where there are more gradual changes, they tend to be at moments where fewer than half of the voices are silent. *Shaar* seems to have more in common with *Syrmos*, and *Voile* with *Aroura*, but in both cases, the density of transitions are much more sparse.

(SEE FIGURE 13) Waiting counts occur at the intersection of a voice with itself, and this diagonal of values indicates the likelihood that the number of silent voices will remain the same in the next transition. In general, a silence count is quite likely to persist, but there are a few exceptions. In *Aroura*, for example, when eleven of its twelve instruments are silent, it's more likely that in the next half-measure it will transition to all voices silent or to only one voice silent. These transitions are equally likely occurrences in *Aroura*. *Syrmos* and *Aroura's* waiting counts are high at each end of the intensity spectrum. *Shaar* and *Voile* are much less distinct in this regard, but the “gaps” in the distribution, places where a certain number of silences never persist, are quite striking.

(SEE FIGURE 14) Another way of looking at the transitions is over the long term, with respect to their stationary probabilities. Xenakis himself spent a good portion of his chapters on “Markovian Stochastic Music” in *Musiques formelles*, analyzing the stationary distributions of *Analogique's* screens and matrix of transition probabilities. There, he commented that “a fundamental criterion of a the evolution of a piece of music can be shaped by the transformations of ataxy over time.” { p. 75 } Computing the stationary probabilities is straightforward with a computer. It involves starting a quantity of automata at various states, and running them through as many transitions as might prove revealing of their temporal behavior. In the illustrations that follow, one hundred automata have been started at each voice quantity, and have been run for ten, one hundred, one thousand and ten thousand transitions. Keep in mind that the data sets of the compositions themselves contain a range of four hundred to seven hundred transitions.

(SEE FIGURE 15) After ten transitions have elapsed, all of the compositions exhibit fairly unstable behavior. Notable in *Syrmos*, *Aroura* and *Shaar* is the dramatic increase in the number of automata with all voices sounding. *Voile* exhibits the same behavior, but the greatest increase is with nine voices silent, followed by all voices sounding. (SEE FIGURE 16) After one hundred runs, the character of the increase in all voices becomes more evident. In *Syrmos*, it reaches a peak and begins to taper off. In *Aroura*, growth de-accelerates, but then continues its climb. *Shaar's* behavior is similar to *Syrmos*, and *Voile* shows continued, rapid acceleration. (SEE FIGURE 17) By one thousand runs, the differences between *Syrmos* and *Shaar* are as great as their similarities, with the initial increase in the sounding of all voices found to be tapering off in *Shaar*. *Aroura's* automata are shown to all be on the increase, and *Voile* exhibits the same behavior such that the simulation's number representation is exceeded. (SEE FIGURE 18) At ten thousand transitions, *Syrmos'* "all voices" automata are still on the increase, and *Aroura* has joined *Voile* in a hyberbolic curve that broke the computer simulation.

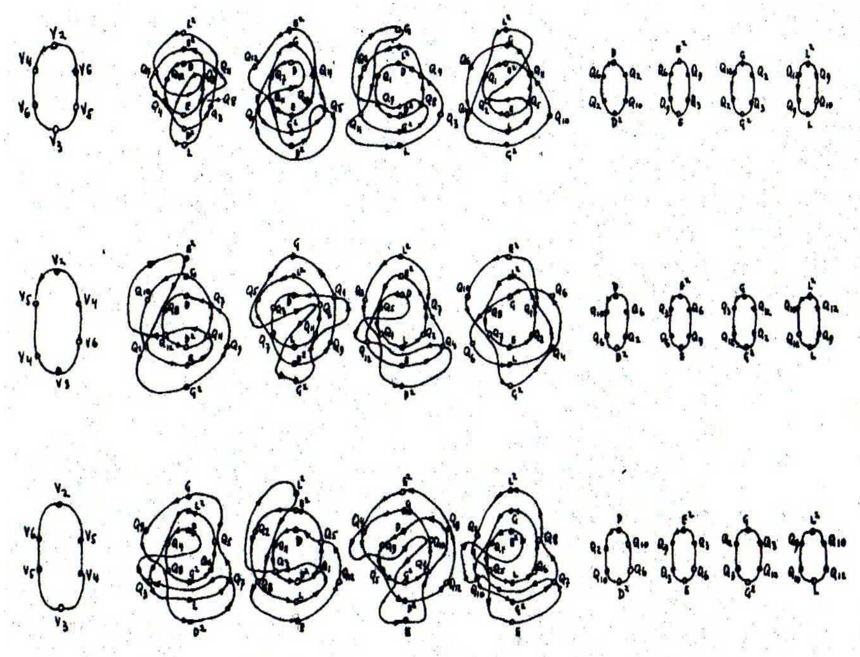
It's worth repeating that because of the generality of the data set, it would be premature to suggest this exploration has strong analytical significance. There's simply not enough information to establish a direct connection to the details of the compositions. But the analysis reflects some correlation with very general features of the pieces, and the representations show differences of character that can't be attributed to some side effect of the the analytical method.

An issue that immediately suggests itself is whether a Markov Chain can adequately describe aspects of Xenakis' later compositions. A work like *Syrmos*, which was composed in part by means of a Markov Chain, suggests analysis of this type to be useful. But what of Xenakis' later compositional methods that are often determinate in nature? Sieves, for example, and cellular automata can also be modeled as

Markov Chains. Further, the basic Markov process demonstrated here could be extended to include memory of past states, or time-variant periods instead of fixed time slices. At issue is whether a statistical view of Xenakis' determinate structures results in an abstraction of the form of his works that prove revealing of their character.

Xenakis used compositional automata throughout his career, but they were mostly fairly simple machines, though often layered at macro and micro structural levels. Xenakis chose to concentrate complexity and non-linearity in his outside-time structures, such as his Sieves, and use simple automata to organize these structures in time. These automata reflect his interest in physics and cybernetics, and seem to exhibit none of the features of planning, or algorithms of symbolic search, that so enchanted the Artificial Intelligence community in the early 1960s. For Xenakis' music was not a language, and he reserved for himself these aspects of compositional decision and intuition.

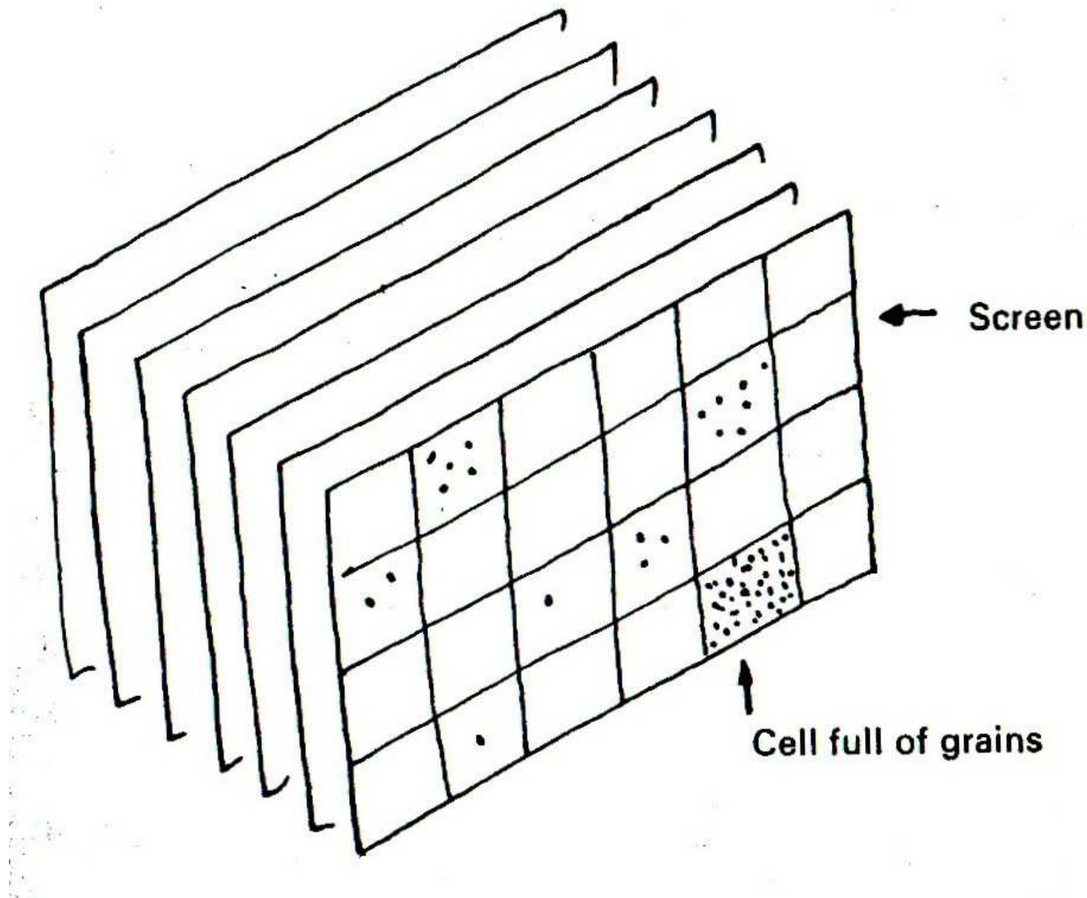
Thank you.



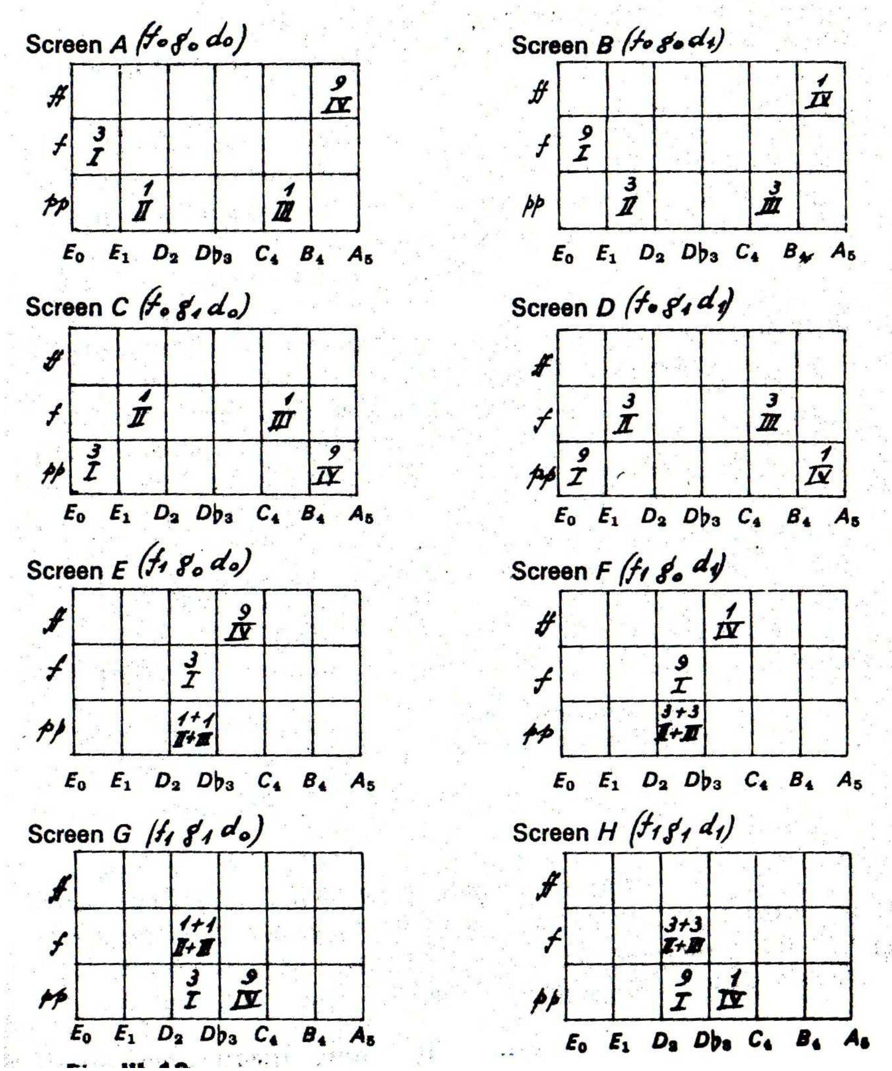
Visualizing Macrocompositional Dynamics in the Works of Iannis Xenakis

Charles Turner
Graduate Center
City University of New York

Marcel Hendrix
Eindhoven Technical University



From Xenakis' *Musiques formelles*: “A book of screens equals the life of a complex sound.”

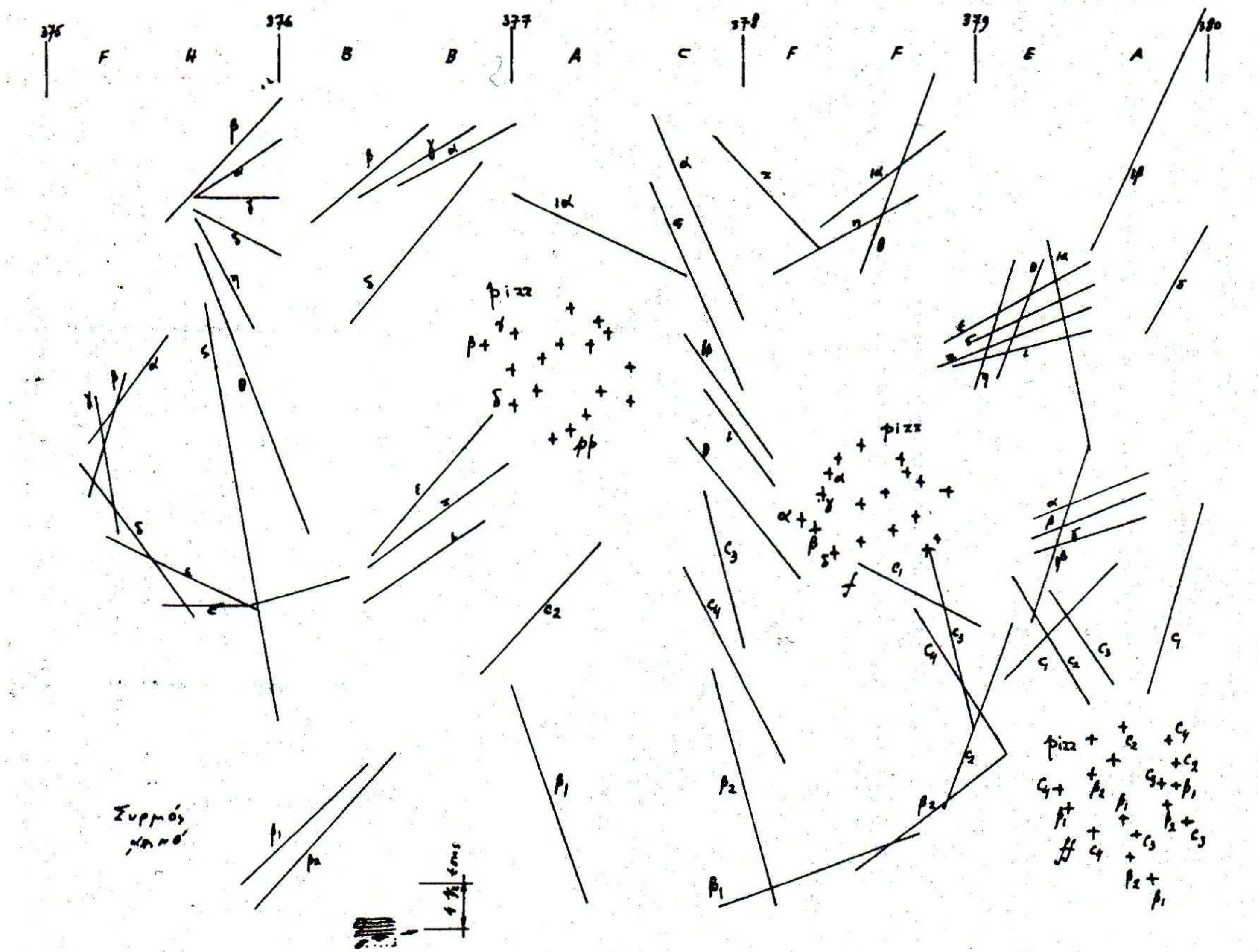


The eight screens of *Analogique A*. The X axis represents pitch,
the Y axis represents intensity, and the Z axis represents sound event density.

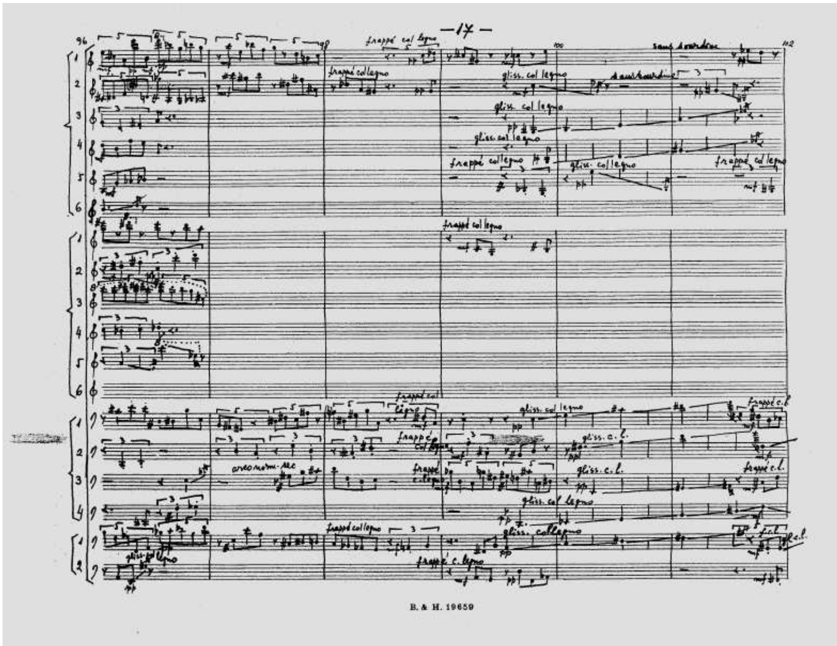
MTPZ

↓	A ($f_0g_0d_0$)	B ($f_0g_0d_1$)	C ($f_0g_1d_0$)	D ($f_0g_1d_1$)	E ($f_1g_0d_0$)	F ($f_1g_0d_1$)	G ($f_1g_1d_0$)	H ($f_1g_1d_1$)
$A(f_0g_0d_0)$	0.021	0.357	0.084	0.189	0.165	0.204	0.408	0.096
$B(f_0g_0d_1)$	0.084	0.089	0.076	0.126	0.150	0.136	0.072	0.144
$C(f_0g_1d_0)$	0.084	0.323	0.021	0.126	0.150	0.036	0.272	0.144
$D(f_0g_1d_1)$	0.336	0.081	0.019	0.084	0.135	0.024	0.048	0.216
$E(f_1g_0d_0)$	0.019	0.063	0.336	0.171	0.110	0.306	0.102	0.064
$F(f_1g_0d_1)$	0.076	0.016	0.304	0.114	0.100	0.204	0.018	0.096
$G(f_1g_1d_0)$	0.076	0.057	0.084	0.114	0.100	0.054	0.068	0.096
$H(f_1g_1d_1)$	0.304	0.014	0.076	0.076	0.090	0.036	0.012	0.144

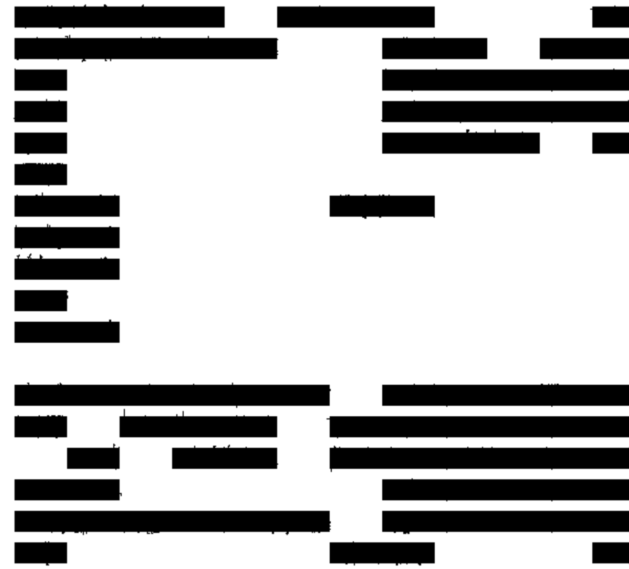
The Matrix of Transition Probabilities from *Analogique A*.
Transitions from screen-to-screen are shown as percentages.



Sketch of *Syrmos* from *Musiques formelles* showing screen succession and measures at top.

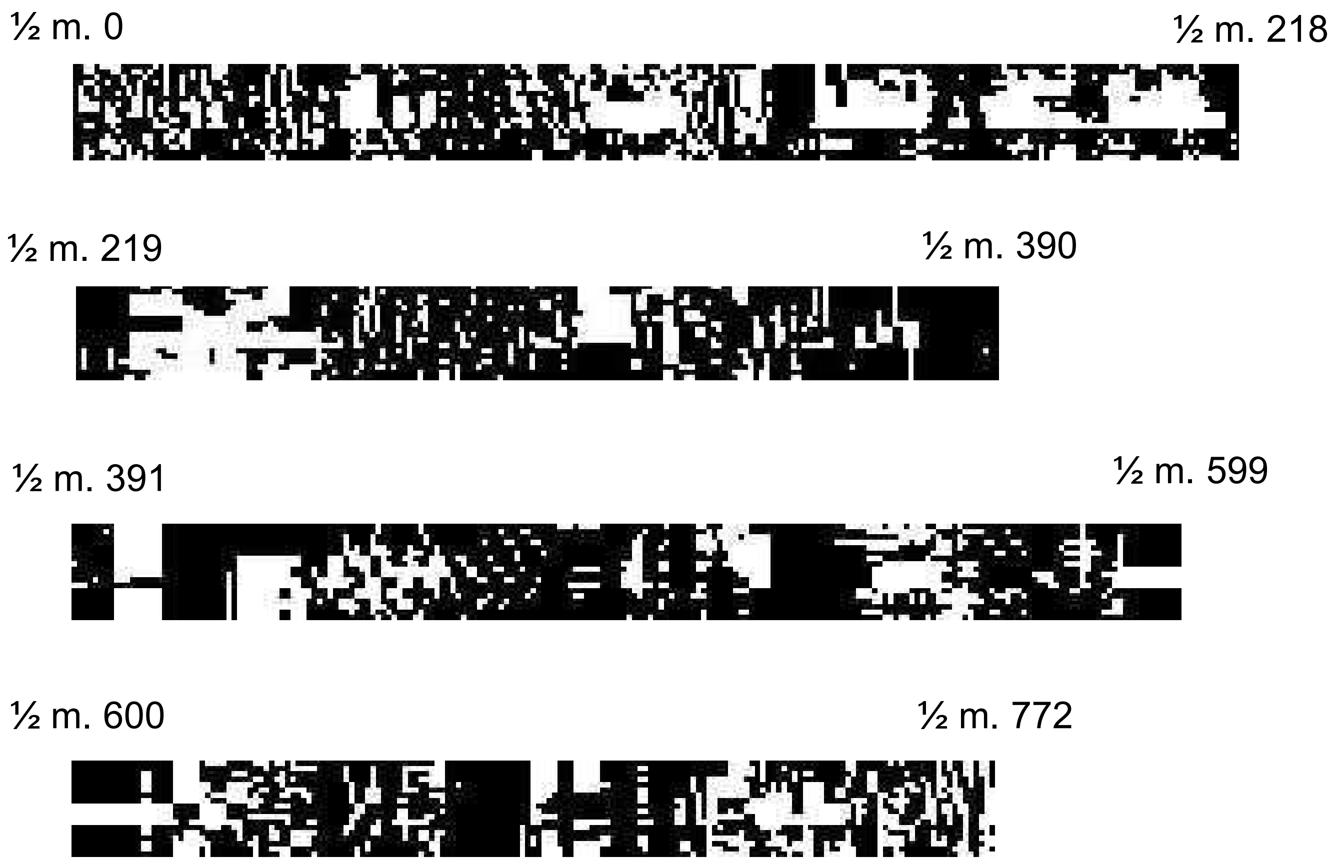


Score



Database
(Silences shown in white)

Translation of a score (*Syrmos*) into a database of silences by voice.
Only rests of one-half measure or greater are catalogued.



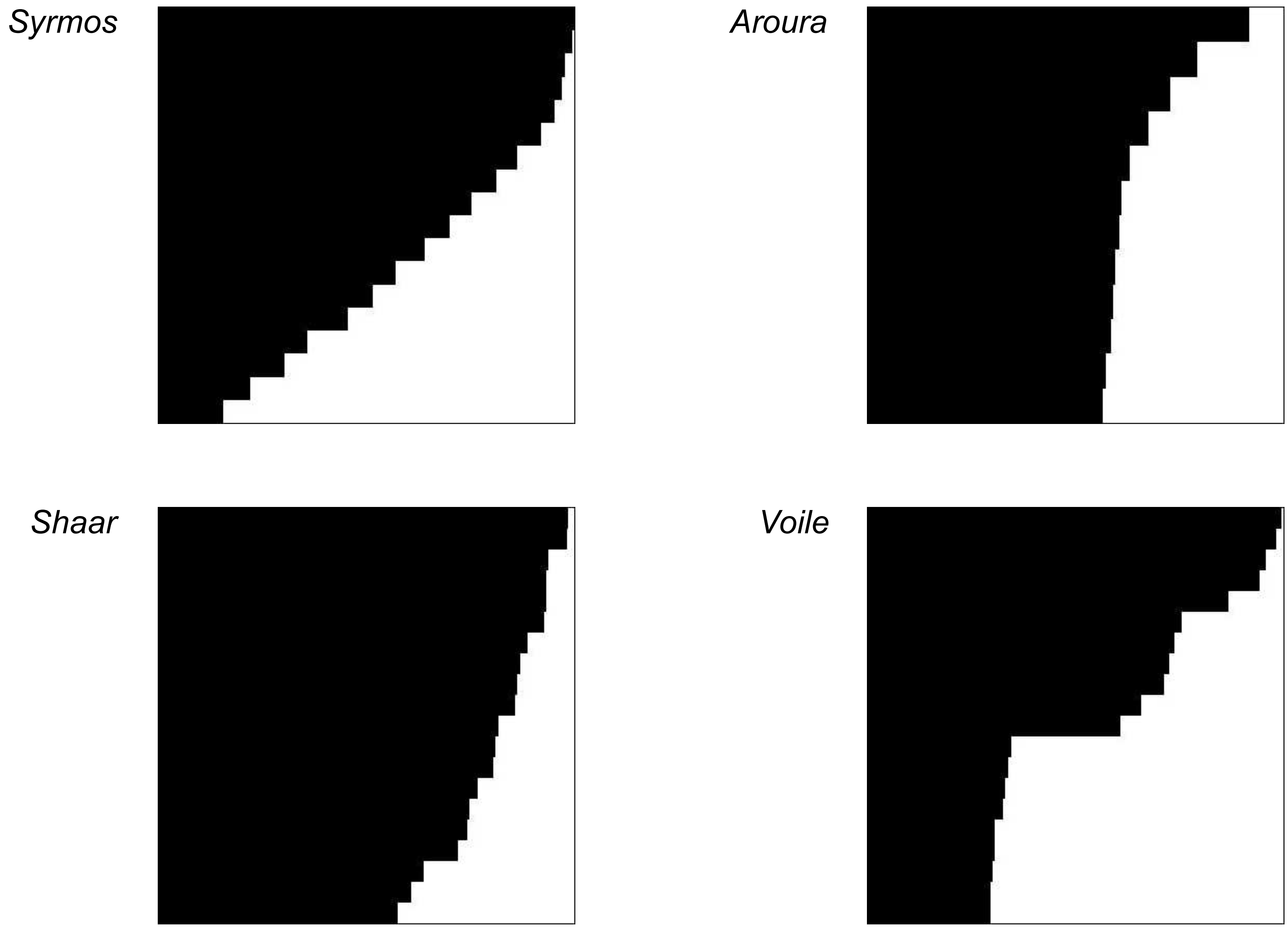
“Silence Map” of *Syrmos* (complete score).
Silences marked in white.

Aroura (complete score)

Shaar (complete score)

Voile (complete score)

Silence maps of *Aroura*, *Shaar*, and *Voile.*
Silences marked in white.



Silence distributions of *Syrmos*, *Aroura*, *Shaar*, and *Voile.*
Silences marked in white.

Current number of silent voices:	Transition to next voice count:	Quantity of transitions:	Percentage of total:
18	16	3	50%
18	17	1	17%
18	18	2	33%
	Total transitions:	6	100%

Construction of the transition table for *Voile* in the case of 18 silent voices.

Voile

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0.83					0.60			0.50	0.03	0.17	0.08				0.07	0.06				
1																					
2				1.00																	
3							1.00														
4																					
5						0.40				0.03		0.08									
6										0.02											
7											0.08	0.08									
8									0.50		0.08										
9	0.04									0.88	0.08	0.08	0.33			0.04	0.06				
10	0.01							0.50			0.05	0.15					0.06				1.00
11	0.01		1.00					0.50			0.08	0.46		0.33	0.33	0.04					
12													0.67				0.06				
13														0.33		0.04	0.06				
14															0.33	0.11					
15	0.04									0.02					0.33	0.63	0.12	0.25			
16	0.01									0.02		0.08		0.33		0.04	0.53	0.25	0.50		
17																	0.06	0.25	0.17	0.33	
18	0.04																	0.25	0.33	0.33	
19										0.02						0.04					
20																				0.33	

The complete transition table for *Voile*.
Axes represent the number of silent voices.

Syrmos

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0	0.64	0.18	0.14	0.10	0.08	0.13	0.07	0.02	0.02	0.05	0.02						0.08		
1	0.14	0.16	0.14	0.07	0.05	0.04	0.02	0.07	0.02		0.04								
2	0.02	0.14	0.25	0.21	0.10	0.13	0.07	0.07	0.07	0.02	0.04		0.05						
3	0.02	0.12	0.16	0.21	0.06	0.11	0.02		0.04		0.02		0.02						
4	0.06	0.08	0.14	0.07	0.45	0.04	0.12	0.06		0.05	0.07		0.05	0.08					
5	0.05	0.06	0.03		0.05	0.11	0.19	0.11	0.11	0.10	0.04	0.03							
6	0.03	0.04	0.03	0.07	0.04	0.07	0.16	0.09	0.07	0.05	0.09	0.08	0.02	0.04	0.07				
7		0.12	0.03	0.10	0.04	0.11	0.07	0.20	0.18	0.05	0.07	0.05	0.09				0.08		
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		
9	0.02	0.02	0.02	0.02		0.04	0.05	0.11	0.04	0.22	0.07	0.15	0.07	0.04					1.00
10		0.04		0.02	0.01	0.07	0.07	0.06	0.09	0.15	0.22	0.13	0.05	0.08	0.07		0.08		
11	0.01			0.02	0.04	0.04		0.02	0.07	0.10	0.13	0.18	0.11	0.21	0.27				
12	0.01		0.03	0.02	0.03		0.02	0.02	0.02	0.07	0.11	0.15	0.30	0.17	0.13	0.17			
13						0.02		0.04	0.07		0.04	0.05	0.11	0.17	0.13	0.33			
14	0.01								0.02			0.08	0.02	0.13	0.27	0.33			
15		0.02			0.01			0.02	0.02				0.02				0.08		
16				0.02									0.02	0.04		0.17	0.62		
17	0.01																	1.00	
18										0.02									

Aroura

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0.89	0.40	0.25	0.50				0.33	0.14	0.06	0.05	0.11	0.23
1		0.20		0.50					0.07			0.01	
2	0.01		0.50						0.03				
3						0.17					0.02		
4					0.25	0.17						0.03	
5	0.01					0.33		0.08	0.07				
6	[illegible]				[illegible]		[illegible]	[illegible]					
7	0.01					0.17	0.25	0.33	0.03	0.03		0.01	
8	0.01				0.25			0.08	0.52	0.13	0.02	0.01	0.04
9	0.01		0.13						0.07	0.65	0.12	0.01	
10	0.01				0.25	0.17			0.07	0.10	0.65	0.03	
11	0.03	0.40					0.25	0.08			0.12	0.70	0.10
12	0.03		0.13				0.25			0.03	0.02	0.09	0.63

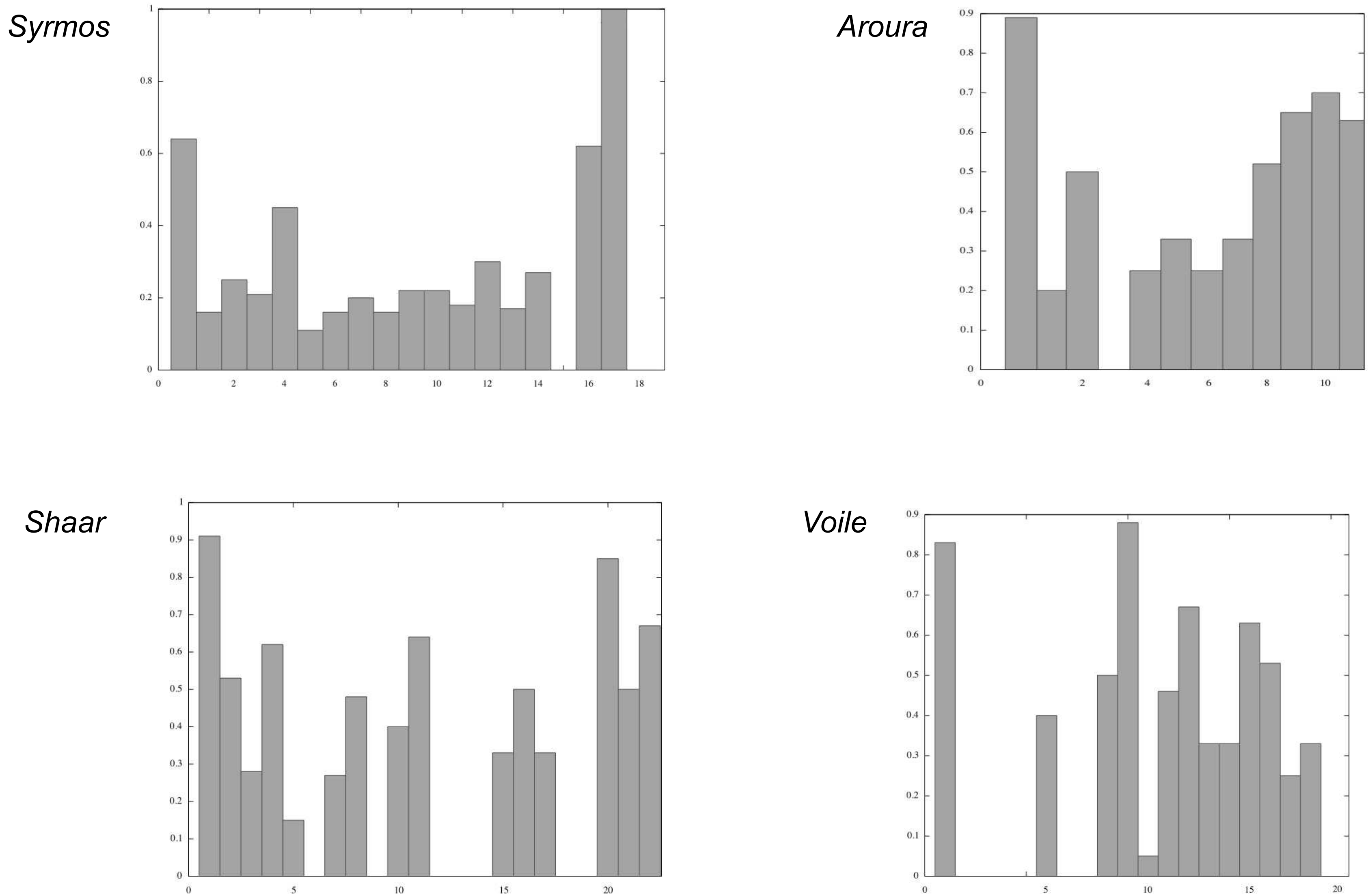
Shaar

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0.91	0.16	0.17	0.11	0.23	0.25	0.18	0.05			0.23			0.11	0.08			0.50	0.12	0.50	0.11
1	0.01	0.53	0.11	0.06										0.11							
2			0.28	0.13	0.15	0.25	0.18			0.20											
3	0.01	0.11	0.22	0.62	0.38			0.05	0.33				0.33								
4			0.11		0.15		0.18	0.19			0.05	0.33									
5		0.05	0.06					0.05													
6	0.01			0.02		0.25	0.27	0.10		0.20								0.50			
7	0.01		0.06		0.08			0.48	0.33	0.20	0.05	0.33									
8		0.05				0.25									0.04						
9				0.02						0.40		0.33									
10	[illegible]							[illegible]			[illegible]				[illegible]						
11													0.33	0.22							
12							0.09		0.33						0.04						
13				0.02							0.05			0.33	0.13						0.11
14	0.01						0.09	0.05					0.33	0.11	0.50	0.67			0.04		
15															0.04	0.33					0.11
16																					
17														0.11							
18	0.01			0.02											0.04				0.85		
19																				0.50	
20	0.01	0.11																			0.67

Voile

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0.83					0.60			0.50	0.03	[illegible]	0.08				0.07	0.06				
1																					
2				1.00																	
3							1.00														
4																					
5						0.40				0.03		0.08									
6										0.02											
7											[illegible]	0.08									
8									0.50		[illegible]										
9	0.04									0.88	[illegible]	0.08	0.33			0.04	0.06				
10	[illegible]							[illegible]			[illegible]	[illegible]					[illegible]				[illegible]
11	0.01		1.00					0.50			[illegible]	0.46		0.33	0.33	0.04					
12													0.67				0.06				
13														0.33		0.04	0.06				
14															0.33	0.11					
15	0.04									0.02					0.33	0.63	0.12	0.25			
16	0.01									0.02		0.08		0.33		0.04	0.53	0.25	0.50		
17																	0.06	0.25	0.17	0.33	
18	0.04																	0.25	0.33	0.33	
19										0.02						0.04					
20																				0.33	

Transition tables for *Syrmos*, *Aroura*, *Shaar*, and *Voile.*



Waiting counts for *Syrmos*, *Aroura*, *Shaar*, and *Voile.*

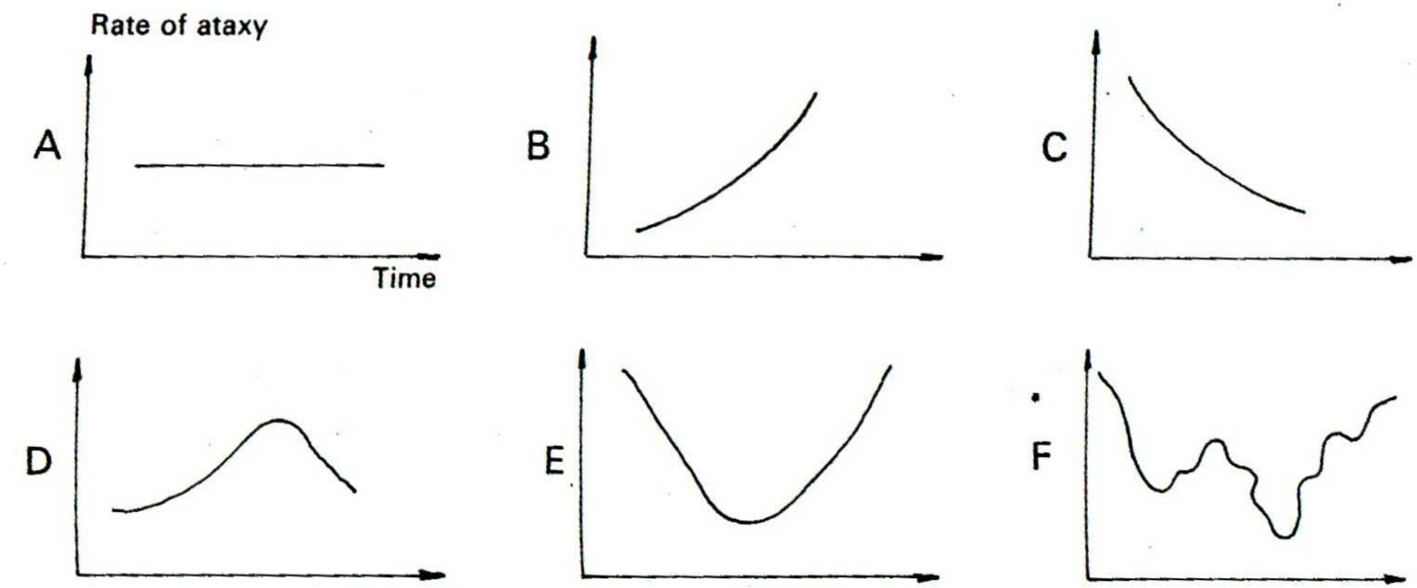
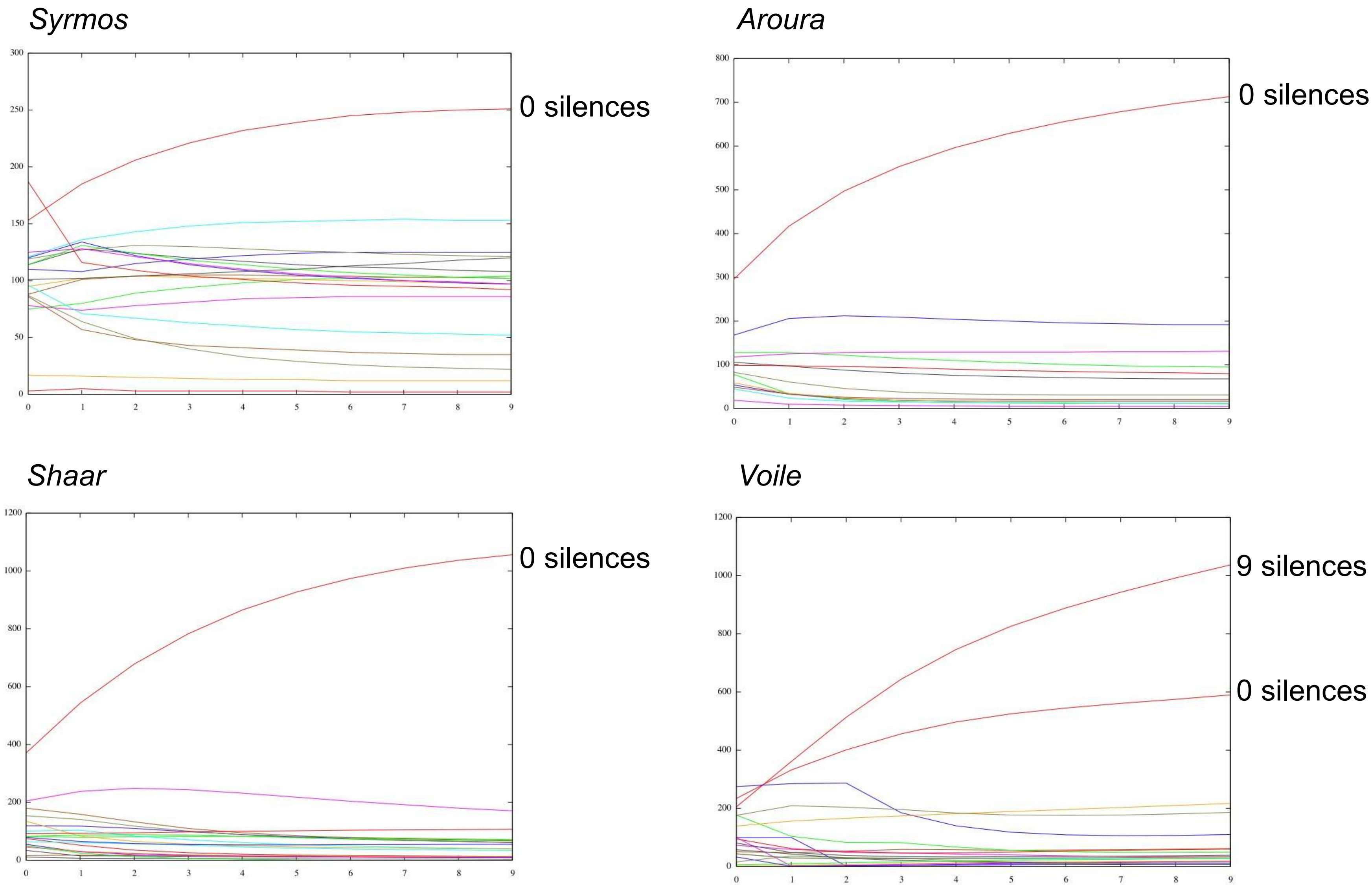


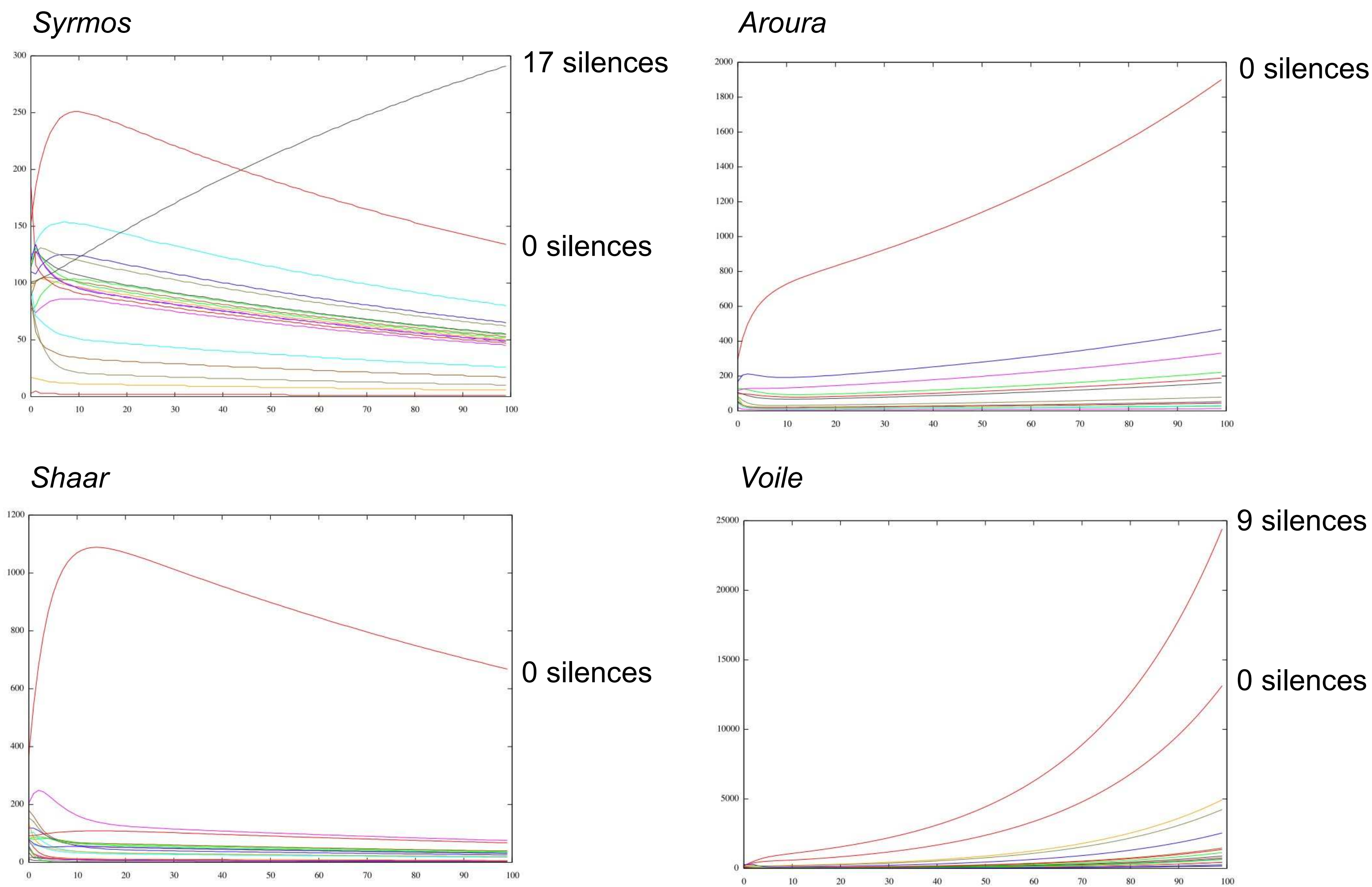
Fig. II–26

A. The evolution is nil. B. The rate of disorder and the richness increase. C. Ataxy decreases. D. Ataxy increases and then decreases. E. Ataxy decreases and then increases. F. The evolution of the ataxy is very complex, but it may be analyzed from the first three diagrams.

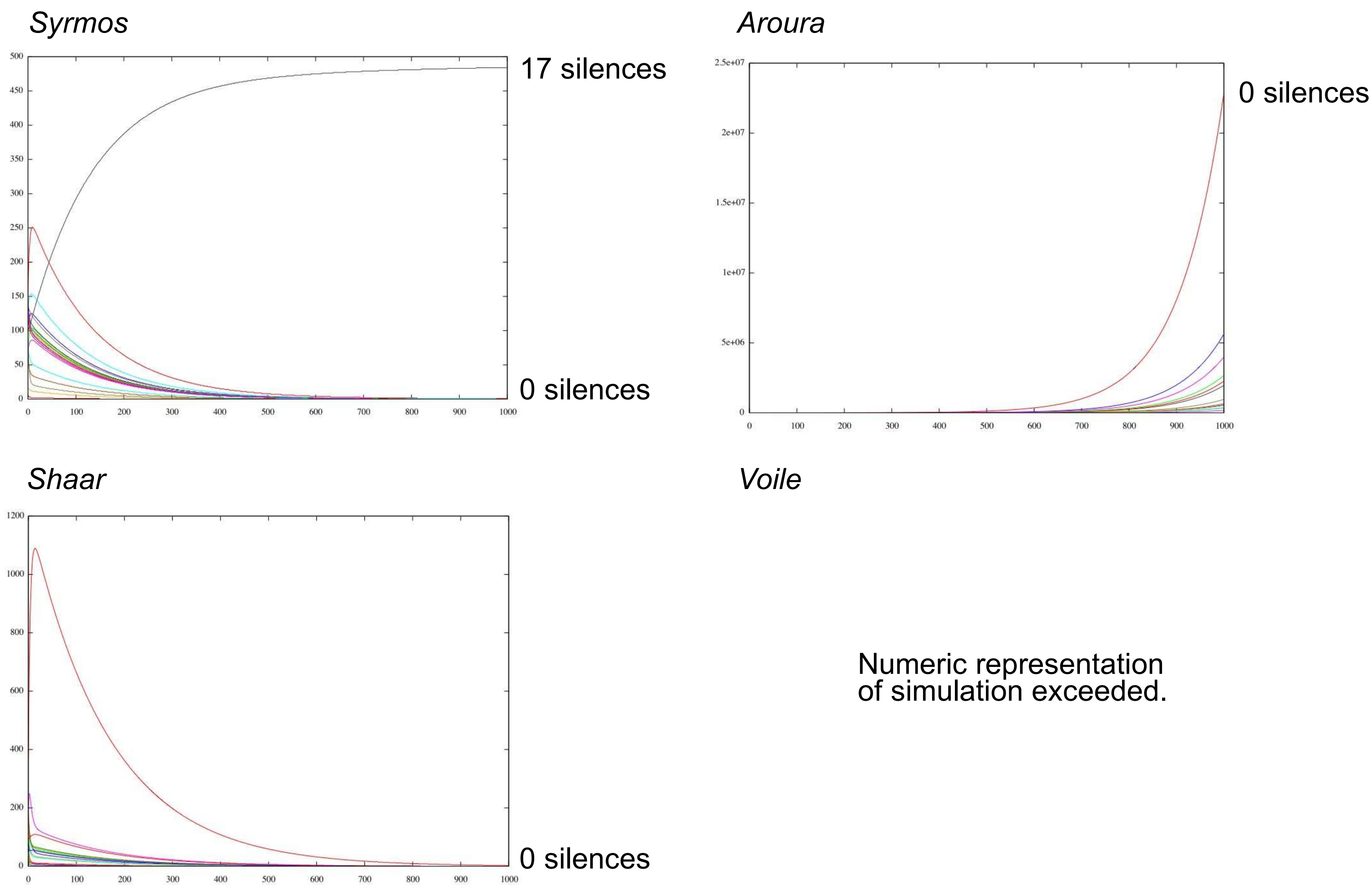
Xenakis' graphs of stationary probabilities from *Musiques formelles*.



Stationary probabilities for *Syrmos*, *Aroura*, *Shaar*, and *Voile* (10 runs).

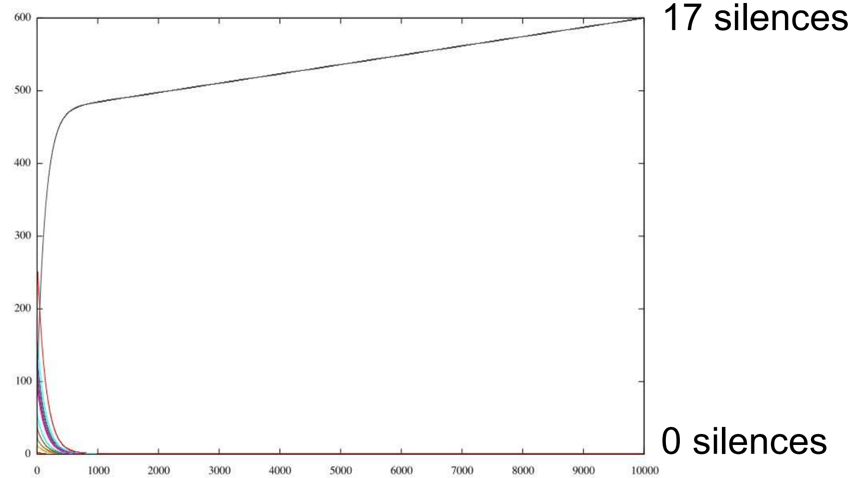


Stationary probabilities for *Syrmos*, *Aroura*, *Shaar*, and *Voile* (100 runs).



Stationary probabilities for *Syrmos*, *Aroura*, *Shaar*, and *Voile* (1,000 runs).

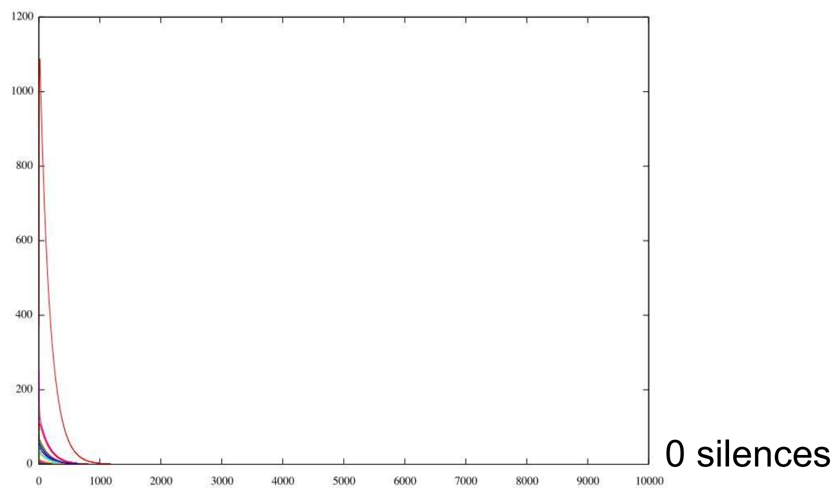
Syrmos



Aroura

Numeric representation
of simulation exceeded.

Shaar



Voile

Numeric representation
of simulation exceeded.

Stationary probabilities for *Syrmos*, *Aroura*, *Shaar*, and *Voile* (10,000 runs).