A Theory of Functional Harmony

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The harmonic series

The musical tone produced by the strings of musical instruments is a complex sound of many frequencies. Stationary nodes that divide the string into vibrating sections are automatically produced at regular intervals along a string. Nodes are also produced in wind instruments but they are easier to visualise in strings. Because a string vibrates between two fixed ends the nodes are constrained so that only an integer number of half-waves can be formed so that a string with good harmonicity vibrates simultaneously as a whole and in two, three, four etc. equal parts. The frequency of vibration is inversely proportional to the vibrating length so the string produces a sound with a set of frequencies that the science of acoustics has shown to be in agreement with the mathematical harmonic series. The harmonic series is both a physical phenomenon and an abstract precise mathematical object. The mathematical expression of the ideal harmonic series is given by:

$f, 2f, 3f, 4f \ldots$ where the terms of the series are ascending integer multiples of the fundamental frequency f.

Our ear mechanism transmits the vibrations of a sound to the basilar membrane which is tapered along its length with a variation in properties so that it progressively resonates to increasing frequencies. Because it is physically continuous, vibration at one position will deflect adjacent parts along its length. Vibrations that have frequencies sufficiently close, but not close enough to merge, will overlap on the basilar membrane and create interference. Because of this physical limitation Beament (2005, p51) says that the 7th harmonic and higher, typically create buzz or hiss. The importance of the first six harmonics to the psycho-acoustic system (PAS) is illustrated by Pierce (1992, p95) observing that when the accurate 7th, 9th and 11th harmonics of a series were presented experimentally, the PAS processed them as the 4th, 5th, and 6th harmonics of another series with a higher fundamental pitch, even with the errors of -2.78%, 0.00% and +1.85%. The frequencies that stimulate the basilar membrane are converted to streams of periodic neural pulses. These pulse streams are then processed neurologically by our PAS to produce the sensation, among others, of the fundamental pitch.

As the harmonic number increases the amplitudes of the harmonics of a musical tone tend to diminish progressively. The harmonics greater than the 6th or 7th may not be clearly distinguished but Beament (2005) does not exclude higher harmonics from contributing to the sensation of the fundamental pitch. When the pitches of higher harmonics of a series are sounded with the strength of a musical tone, or low harmonic of another series, they will be distinguished. When the first inversion of a triad, such as EGC, is sounded the pitches of the musical tones correspond to the 5th, 6th and 8th harmonics of the C harmonic series. The C will be distinguished as there are no harmonics generated by the sounded musical tones that cause interference on the basilar membrane. The first six harmonics are sufficient to give the sensation of a musical tone with a strong fundamental pitch. For the musical tone C, for example, these are CCGCEG, in ascending order. It is the relationships between the first six harmonics of the harmonic series that is the basis of the theory of harmony presented here.

When multiple musical tones are present in a sound, the PAS uses heuristic shortcuts to decompose it quickly and into separate sounds rather than using a perfect mathematical analysis (Bregman, 1990). It relies on rules of thumb such as the sequential and simultaneous grouping of frequencies related by the harmonic series, and assuming that sounds that start more-or-less at the same time have the same source. When a musical tone is sounded the PAS will detect a common time of initiation for the harmonics. Because a sufficient number of wave peaks or neural pulses are required to give the PAS a stable indicator of periodicity our perception of the harmonics will stabilise in descending order until the terminal stability of the fundamental is acquired. Plomp (1964) showed that temporal integration for pitch depends on frequency, low frequencies require more cycles for stable perception. The PAS experiences a temporal evolution of pitch from dispersion to consolidation, an assimilation from spectral multiplicity to tonal unity. Following a postdictive process whereby the fundamental might be assumed by our PAS to be a consequence of the previous sequence of related harmonics, a high probability predictive process and expectation of finality could have developed over time through repeated confirmation.

Beament (2005, p46) says that the PAS is tolerant of imprecision and temporal variation because they are inherent in real sounds. The 12-tone equally-tempered scale (ETS) has an imperfect match with the pitches of the ideal harmonic series, although some harmonics are closely approximated. The PAS appears to process pitch and harmonic relationships as if it has developed a biologically efficient loose-fit template, based on the harmonic series, that is sufficiently effective for us to perceive musical tones, melodies, triads, and harmonic progressions as coherent phenomena.

Just as the harmonic series can be mathematically defined the abstract sub-harmonic series is given by:

f, ***f/2***, ***f/3***, ***f/4* …** where the series is created by integer division of the fundamental frequency ***f***.

Every 2nd wave peak of ***f***, will coincide with every peak of ***f/2***, every 3rd wave peak of ***f***, will coincide with every peak of ***f/3*** so the n^{th} wave peak of ***f*** will coincide with every wave peak of the n^{th} sub-harmonic. A pure tone harmonic, therefore, has within its sinusoidal wave-form all of its sub-harmonics and can theoretically fit onto and reinforce every wave peak of any of its sub-harmonics and the corresponding periodic neural pulse stream. This is the reason for the assimilation of the harmonic series to the fundamental pitch: the fundamental is the common sub-harmonic of all of its harmonics. Every harmonic can potentially reinforce every wave peak and periodic pulse stream of the fundamental. Conversely, the fundamental of the harmonic series does not contain any of the higher harmonics, they are not generated by the fundamental: they are generated by the string.

The harmonic series is a self-similar structure: higher harmonic series are nested within any given harmonic series. Every harmonic is a sub-harmonic of those harmonics that are integer multiples of itself. For example, the 3rd harmonic is a sub-harmonic of the 6th, 9th,12th… harmonics. It is conceivable that the PAS could apply a learned process of harmonic assimilation to the nested series. For example, the 6th harmonic could be assimilated to the 3rd harmonic, as an intermediary sub-harmonic, before the 3rd was finally assimilated to the fundamental. This could be simpler and more efficient than remembering every harmonic until the fundamental is established. Balsach (2020, p33) is clear on this, he says that a harmonic with an odd prime number, n, assimilates the higher harmonics, 2n, 3n, 4n etc.. Odd prime harmonics would then be assimilated directly to the fundamental.

The 1st, 2nd, 4th… harmonics cannot be assimilated to an odd prime harmonic. They form a nested octave series of the fundamental frequency. Each of these octave harmonics is the common sub-harmonic of all the higher octave harmonics. These octave harmonics will also be subject to a nested assimilation process, the final step being from the prime numbered 2nd harmonic to the fundamental. The pure tone fundamental contains the latent 2nd, 4th, 8th… sub-harmonics as lower octaves. This harmonic to sub-harmonic continuity of nested octaves may contribute to the perception of octave equivalence. Balsach (2020, p14) says that the auditory system identifies a musical tone with its octave because of the many harmonics in common. Parncutt (1989, pp47,62) says that musical tones exhibit octave ambiguity. Octave equivalence enables the simplicity of treating pitch classes and their relationships without the need to consider register.

The Harmonic Terrain

When extrapolated the harmonic relationship between the 3rd harmonic and the fundamental leads to twelve pitch classes that are approximated by the 12-tone ETS shown as the circle of fifths in Figure 1. On the circle of fifths the direction of harmonic to sub-harmonic assimilation is anti-clockwise. This directional assimilation from harmonic to sub-harmonic is structural and not a temporal cause-and-effect process. Because the twelve tones form a cyclic group the harmonic relationship between pitch classes is invariant in transposition: choosing to use one pitch class as a reference datum in an example does not compromise the generality of any conclusions.

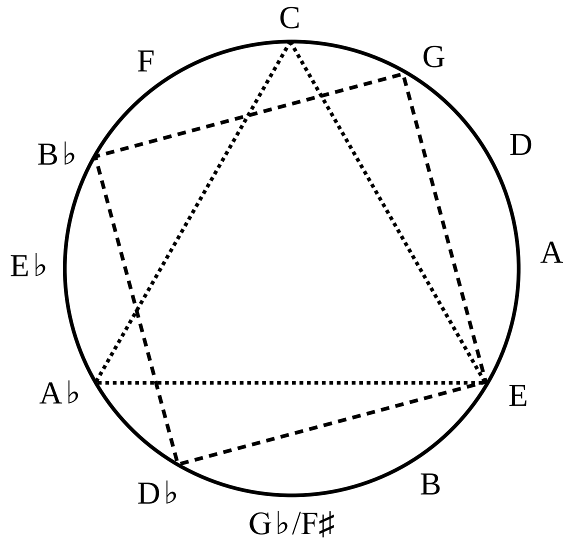

Figure 1.
Circle of fifths with triangular and square groups.

To make the following discussion concrete C is chosen as the reference fundamental of a harmonic series and the 3rd and 5th harmonics will therefore have pitch classes G and E respectively. The harmonic relationship between C and G produced the 12-tone ETS. In the harmonic to sub-harmonic direction the relationship between E and C produces the cyclic group:

E, C, (A♭/G♯), E.

This group contains three distinct pitch classes, shown as a triangle in Figure 1. Enharmony is implicit with the ETS. The relationship between E and G produces the cyclic group:

E, G, (B♭/A♯), (D♭/C♯), E.

This group is shown as a square in Figure 1. Transposing this cyclic relationship to C and G♯ in the triangular group produces the cycles:

C, (E♭/D♯), (G♭/F♯), A, C and (G♯/A♭), B, D, F, (A♭/G♯).

These three groups of four pitches, raised on the three pitches of the group of three, contain all twelve pitches of the ETS. When the three groups of four are aligned according to the group of three a 4 x 3 array is formed. Repeating this elementary cyclic array of twelve pitches creates the Harmonic Terrain, a *tonnetz*, shown in Figure 2. The columns of the Harmonic Terrain are cycles of minor thirds and contain the pitches of the three distinct diminished seventh chords. The rows are cycles of major thirds and contain the pitches of the four distinct augmented triads. The Harmonic Terrain is the mirror image of a *tonnetz* used by Holland (1989) in the development of a computer-based teaching aid. He noted that the '"thirds space" representation is intimately connected with the way in which people perceive and process tonal harmony' and that it is the most compact two-dimensional arrangement of twelve tones.

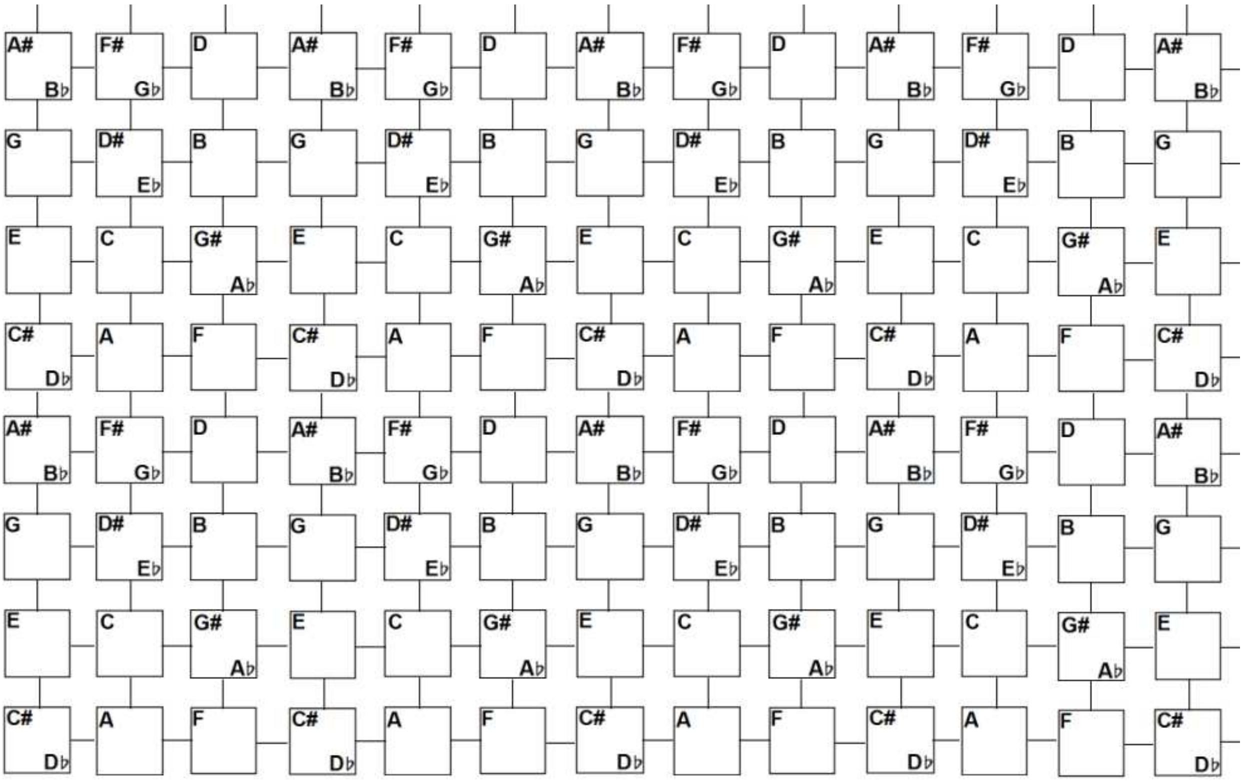

Figure 2 Harmonic Terrain

The function cycle **TSD** runs from left to right.

Each major triad forms a simple L-shape on the Harmonic Terrain and the major seventh chord is a vertical extension of this. Using C as a reference pitch class, the column immediately to its left contains the pitch class group E, G, (B♭/A♯), (D♭/C♯). These correspond approximately to the prime harmonics 5^{th}, 3^{rd}, 7^{th}, 17^{th} and 5^{th} of the harmonic series with C as fundamental. Using octave equivalence this could be rendered as 10^{th}, 12^{th}, 14^{th}, 17^{th} and 20^{th} which makes the cyclicity more apparent. Due to invariance of transposition these four pitch classes, taken in the appropriate cyclic order, are also the pitch classes of the 5^{th}, 3^{rd}, 7^{th} and 17^{th} harmonics of each of the pitch classes in the column containing C. This harmonic relationship is also invariant when transposed to any of the twelve pitch classes. Conversely, all of the pitches in the column containing C are the sub-harmonics of all of the pitches in the left hand column, also invariant in transposition.

Prime or relatively prime harmonics such as the 6^{th} and 5^{th} do not have a harmonic – sub-harmonic relationship. When the fundamental it is not sounded every 6^{th} wave peak or neural pulse stream of the 6^{th} harmonic can fit onto every 5^{th} wave peak or pulse stream of the 5^{th} harmonic to produce a salient wave peak or pulse at the frequency of the fundamental known as a virtual pitch. In the 1970s and 80s Terhardt (1974) developed theories of virtual pitch and harmony that extended Stumpf's (1911) concept of tonal fusion. The periodicity of virtual pitches can be detected electrically from neural pulses and illustrates how strong a drive there is within the PAS to find a sub-harmonic that gives a final sense of pitch. If the pitches G and E corresponded to the 6th and 5th harmonics, then a virtual fundamental, C, would be created. It is seen on the Harmonic Terrain that the G and E pitches can also serve as the 7th and 6th harmonics of A, due to the loose-fit tolerance of the PAS, so a virtual fundamental A could also be produced.

Terhardt (2026) in his *Harmony* section of *Topics of Research in Retrospect* demonstrated an algorithm for finding the roots of chords. This was based on the principle “that all candidates for roots must be subharmonics of the spectral pitches elicited by the actual sound, and that the prominence of any root is enhanced by "subharmonic coincidence".” He used the 2nd, 3rd, 5th, 7th and 9th sub-harmonics in his algorithm, confirming the importance of sub-harmonics in harmony. Not using the 9th sub-harmonic in his two examples would have made no difference to his predicted roots. The Harmonic Terrain, perhaps, offers a simpler visual way of identifying sub-harmonics and chord roots, and potential routes for harmonic progression. The harmonic theory proposed here suggests that only the prime sub-harmonics, 2nd, 3rd, 5th and probably the 7th, should be used. The fundamental is the sub-harmonic to all of its harmonics and this relationship in the lower part of the harmonic series will have been experienced by the PAS.

The Harmonic Terrain is derived from the structure of the harmonic series. Only the relationships between harmonics 1, 3, and 5 were used but the odd prime harmonics 5, 3, 7 and 17, appear in the columns of the Harmonic Terrain. The odd prime harmonics 11 and 13 are notably absent.

Table 1 shows the prime harmonic numbers 3 to 17. Comparing the nearest tones of the 12-tone and 24-tone equally-tempered scales to these prime harmonics shows discrepancies. For the 3rd, 5th and 17th harmonics the 12-tone ETS gives the much better fit. For the 11th and 13th harmonics the 24-tone ETS (the quarter tone chromatic scale) gives the much better fit so it appears that the 11th and 13th harmonics may not be in sufficient agreement with the 12-tone ETS. The 7th harmonic is not fitted well by either the 12- or 24-tone ETS. With C as fundamental the nearest tone to the 7th harmonic on the 12-tone ETS is B♭. When B♭ is added to the C major triad the C7 chord is formed, traditionally described as dissonant, perhaps due to a sense of harmonic strain. However, B♭ contributes essential good harmony in the E♭ and G♭ triads. Overall, any disadvantage appears to be outweighed by the benefits.

The 17th harmonic is fitted well by D♭ and so could theoretically be assimilated to C. The simultaneous sounding of D♭ and C, in close order produces a dissonant sound due to strong mutual interference on the basilar membrane but when sounded sequentially this is avoided.

Table 1. Percentage errors of 1/2 and 1/4 tones for odd prime harmonics.

Harmonic Number	3	5	7	11	13	17
1/2 tone error %	-0.11	0.79	1.82	2.85	-2.31	-0.29
1/4 tone error %	2.81	-2.08	-1.08	-0.08	0.55	2.64

Functional harmonic progression

The process of direct assimilation of harmonic to sub-harmonic is proposed as the prototype for functional harmonic progression observed in music. It follows that direct functional harmonic progression can only take place from the pitch classes in one column of the Harmonic Terrain to the pitch classes in the column to its immediate right.

Although arbitrary labels could be used, in the interest of continuity the Riemannian and traditional convention of using Tonic (**T**), Subdominant (**S**) and Dominant (**D**) to label functions is adopted. The three functions, however, are determined solely by the harmonic relationship between pure tones predicated by the structure of the harmonic series. Function is seen to be an inherent potential for harmonic to sub-harmonic assimilation in the pure tone pitch class space defined by the 12-tone ETS; it is an emergent relative property of pitch class.

Assigning **T** function to the pitch class of the fundamental of a harmonic series implies that the 3^{rd} harmonic, by definition, belongs to the pitch class of the dominant and so has **D** function. It follows that the pitch classes of the 3^{rd}, 5^{th}, 7^{th} and 17^{th} harmonics all have **D** function because they have a direct harmonic to sub-harmonic relationship with the **T** fundamental. Conversely, the 3^{rd}, 5^{th}, 7^{th} and 17^{th} sub-harmonics of the fundamental must all have **S** function.

With **T** function C as a reference tonic, E will have **D** function. E also has **D** function with a **T** function A. It follows that A must have, and will always have, the same function as C because they will always have the same functional relationship with E. That C and A must always have the same function is a relationship that is invariant with transposition so that all pitch classes in a column must always have a common function.

The columns of the Harmonic Terrain can then be labelled from left to right, relative to a tonic, revealing the structural function cycle **TSDT**. The function cycle is not inferred from composed musical syntax, it is deduced from the structure of the harmonic series.

All the pitch classes in a column of the Harmonic Terrain have the same function but the PAS may have learned to assign differential significance associated with the relative amplitudes of the harmonics of musical tones and their closeness to the fundamental. The order of descending strength and significance is suggested as 2^{nd}, 3^{rd}, 5^{th}, 7^{th} and 17^{th} harmonics. With C as the reference pitch this would be C, G, E, B♭ and D♭.

The inherent assimilative relationship between pitch classes is independent of any tonic and is invariant in transposition. Harmonic assimilation is a process predicated by the inherent structural relationship between pure tones of the harmonic series and how they are processed by the PAS. Pure tones are the sinusoidal quanta of musical sound, there is no deeper level in musical sound at which a prototype for harmony can be found.

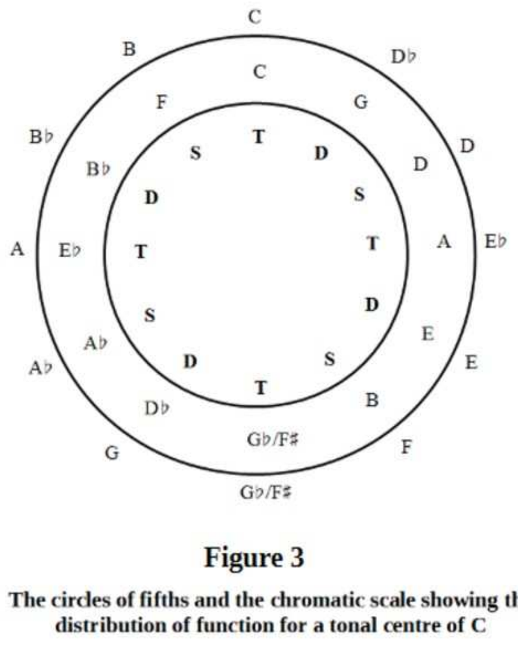

Figure 3

The circles of fifths and the chromatic scale showing the distribution of function for a tonal centre of C

Figure 3 shows the result of assigning function to the tones on the circle of fifths with C as tonic. Lendvai (2015), among others, shows the same allocation of function on the circle of fifths. Rearranging the pitch classes into the close-order cycle of minor seconds, shown in the outer circle of Figure 3, reveals that the descending chromatic scale is also arranged in functional order.

The Harmonic Terrain has the following characteristics:

(a) The main diagonals descending from left to right contain the pitches of the circle of fifths in functional cycles.
(b) The main diagonals ascending from left to right contain the pitches of the chromatic scale, in functional cycles.
(c) The steeper diagonals, ascending (and descending) contain the pitches of the two distinct whole-tone scales ascending in functional cycles.
(d) The rows contain the pitches of the four distinct augmented triads in functional cycles of major fourths.
(e) The columns contain the pitches of the three distinct diminished seventh chords in equifunctional cycles of minor thirds.

All of these cyclical musical objects appear as straight lines on the Harmonic Terrain and as regular polygons on the circle of fifths. Ball (2011, p197) produced an array on which these cycles can also be traced as straight lines. The Harmonic Terrain also shows agreement with the topographical relationship between major and minor keys observed in the psychological research of Krumhansl and Kessler (1982).

Functional harmonic progression is simply the assimilation of one pitch to a sub-harmonic pitch. It seems reasonable that the PAS would use the nearest available prime numbered sub-harmonic. That the nearest sub-harmonic is used is supported by the results of Parncutt and Radovanovic (2022) who analysed 7,415 cases of the suspended triad. For example, with CFG as the suspended triad, F would be the most likely root, well ahead of G and C. In this case CFG would correspond to the 6th, 8th and 9th harmonics of the harmonic series with F as fundamental. The prototypical process of nested assimilation or harmonic progression will apply to pure tone harmonics below the surface of the score as well as to the fundamentals of musical tones and the roots of chords. Harmonic progression from chord to chord is essentially structural and is temporal only in the sense that the score is temporal. Progression can also occur within the duration of a single musical tone or chord where harmonic processes occur over a shorter period. Harmonic progression within a major triad implies that the three musical tones would be processed as nested series to a single fundamental or root whether they are sounded simultaneously or spread; we have short, medium and long term memory for pitch.

Functional diaharmonic progression

Functional harmonic progression is not restricted to progression between the fundamentals of scored musical tones or roots of chords. Every musical tone is sounded with its harmonics that are always present below the surface of the written score. Therefore, functional harmonic progression during music can occur between the fundamental of one musical tone and a harmonic of another musical tone, including those that would not be identified as the root of a chord. We may agree that a clear and unambiguous root may be assigned to some chords but, even so, it is logically mistaken to assume that harmonic progression occurs only between the roots of chords. The score specifies what notes to play but musical notation is only a shorthand version of what we hear and process.

Every musical tone has two functions within it. It is heard as a lower harmonic series and processed using functional harmonic assimilation or progression. Where the fundamental pitches of a chord's constituent musical tones range over more than one column of the Harmonic Terrain pitches of all three functions will be present. Diminished chords contain two functions. Functional harmonic progression occurs within chords.

In the melodic sequence B-C each musical tone is sounded as a harmonic series and consequently has two functions, one attached to the fundamental pitch and the other to the odd prime harmonic pitches. (It may be useful to to follow this discussion on the Harmonic Terrain, Figure 2.) With C as tonic the function sequence for B-C on a score would be **S-T** but, because each tone has two functions, the function sequence processed is (**T-S**)-(**D-T**). The **S** function fundamental B pitch can progress functionally and harmonically to the **D** function harmonics, E and G, of the fundamental pitch C. This combined sequence of functional harmonic progressions is designated functional diaharmonic progression, "dia" used in the sense of "through." Functional diaharmonic progression takes place within our PAS between musical tones in non-adjacent columns of the Harmonic Terrain in the direction of the function cycle **TSDT**. This progression uses the pitches of the lower prime harmonics of the scored second tone as functional intermediaries. Analysis of harmonic progression in a score must consider the harmonics of scored musical tones.

The intermediary pitches can also be the fundamentals of musical tones. For example, in a plagal cadence of major triads the roots are unambiguous and have the functional progression **S-T**. This is seen as functional diaharmonic progression, **S**-(**D-T**). For example, with the triad FAC followed by the triad CEG the **S** function F can progress functionally to the **D** function fundamental of the musical tone G as an intermediate pitch to C. The structural order of function is preserved but not directly from triad root to root.

Consider the major triad CEG preceding the chord GBDF. G is not a direct sub-harmonic of C so the C root cannot directly progress functionally to G. The C triad will be processed to produce a strong fundamental C pitch that can progress functionally to the fundamentals of the F and D musical tones because they are direct sub-harmonics of C. The F and D pitches are present as strong fundamentals of musical tones which can then progress functionally to G. If F is not scored and is present only as the 7^{th} harmonic of G it may not be distinguished by our PAS. If the D is also not scored it will still be present as a strong harmonic of the musical tone G. Some of the pitch classes of F and D will always be present as musical tones or harmonics, enabling functional diaharmonic progression. Functional diaharmonic progression dispenses with the need for the notion of functional regression for which there is no prototype.

The six-four (6/4) chord is a second inversion of a tonic triad and occurs in the cadential sequence **T-S**-(6/4)-**D-T**. There has been theoretical discussion and controversy regarding the function of the 6//4 chord (Damschroder, 2008, pp31,43) but not about the following **D** chord. With functional diaharmonic progression the sequence is seen to be **T-S**-(**D-T**)-(**S-D**)-**T**. The 6/4 chord is entered using a **D** function pitch and left using the **T** function pitch of the root. The following chord is then entered using an **S** function pitch and left using the **D** function pitch. This sequence contains two diaharmonic progressions and the essential cyclic order of function is preserved throughout. Schenker (1980, p229) saw the interpretation of the six-four chord as an inverted tonic triad as lacking in artistry. He asked why we would disrupt the beauty of the cadence with a tonic chord. He said that the cadential six-four chord could only be analysed with **D** function. Schenker's assertion does not explain anything but he obviously thought that the function cycle was important, so important as to be decisive. This example emphasises that it can be a mistake to assume that chords have a single function. Harmonic progression does not always take place between the roots of chords even when the roots are clear and unambiguous. In this case recognising the multi-functional structure of chords and diaharmonic progression leads to a coherent explanation, consistent with the function cycle.

In the context of Rameau’s *double emploi*, Gossett, in his translation of Rameau's *Treatise on Harmony* (1722/1971, p71, n[16]), has Rameau saying, of the chords FACD and DFAC, that they “can function in two ways simultaneously, or rather can serve one function at the beginning of the beat and the other at the end.” Damschroder (2008, p271) describes Rameau’s *double emploi* in much the same way. It is clear that Rameau thought that a chord could be entered through one tone and left through another.

Equifunctional isoharmonic progression

The musical tones a minor third apart, such as C and A, can be described as isoharmonic because they have their prime harmonics and sub-harmonics in common. The prefix “iso” is used in the sense of “same.” For example, they have harmonics of pitch E and G in common. Conversely, both E and G have C and A as common sub-harmonics and can, therefore, progress functionally and harmonically to both C and A. C and A have sub-harmonic pitches F and D in common which are potential virtual roots. This inherent sharing of harmonics is why the minor chord ACE(G) is harmonious but with some degree of ambiguity as to its root. This isoharmonic relationship between A and C enables the progression, or switch, from one to the other. This is designated as equifunctional isoharmonic progression. When the minor triad ACE is sounded some ambiguity will be present as to the root even though A is the most salient. This ambiguity has sometimes lead to a preference for the unambiguous major triad to terminate a final cadence.

Tritone pairs, such as C and are also isoharmonic. C and F♯ have harmonic pitches E and B♭/A♯ in common. These harmonic pitches will be distinguished as they will both be sounded as 5th (and perhaps 7th) harmonics. In augmented 6th chords they will be sounded as musical tones. C and F♯ have sub-harmonic pitches in common. They can switch with each other, and progress to the same sub-harmonics. The isoharmonic relationship between C and F♯ may be weaker than the minor third harmony due to the dependence on the strength of 5th harmonics only.

The dominant chord in the minor mode

When the minor triad EGB is sounded before the minor triad ACE the G will have a stronger tendency to assimilate to and reinforce its nearest sub-harmonic, C, rather than A; C is the 3rd sub-harmonic of G while A is the 7th. However, if the major triad EG♯B is sounded then the reinforcement of C will be reduced by the absence of the musical tone G. The musical tone G♯ will assimilate to and strongly reinforce its 3rd sub-harmonic, E, that will then progress to its nearest sub-harmonic, A. While the relative strength of the C is reduced that of the A is increased so tipping the balance in favour of A as the more salient pitch. This explains why the major dominant chord is typically preferred in the minor mode.

The Locrian distension

A particular succession of chords is described by Balsach (2020) as the Locrian distension or relaxation: for example, CEG followed by EG♯B. It would appear that the root of the first triad regresses to the root of the other at a major 3rd distance. However, it is clear from the Harmonic Terrain that functional diaharmony is present in the progression C to A♭/G♯ to E. C does not directly progress, or regress, to E. This is another example where seeing harmonic progression as only occurring between chord roots is misleading and obscures the functional progression. Two consecutive occurrences of the Locrian distension are present in the major chord sequence C-A♭-C-E-C in Rehding (2008, p196). Hugo Riemann said, “This bold but forceful and sweet-sounding succession cannot be defined in the sense of a tonality of the older kind...”

The Phrygian progression

The apparent progression of the major triad D♭FA♭, descending by a semi-tone, to CEG is an example of the Phrygian progression. Even when the D♭ pitch is sounded with the full strength of a musical tone it is unlikely that it would directly assimilate to C because the virtual pitch would be six octaves lower than the sounded C, and the PAS could not have experienced the 17th harmonic assimilating to the fundamental of a harmonic series. There is, however, an alternative harmonic route: D♭ has an isoharmonic relationship with B♭, E and G all of which can directly progress functionally and harmonically to C. B♭ would be produced as a virtual root of D♭FA♭; E and G would be present in the C triad. The Phrygian progression could, therefore, be seen as a combined sequence of equifunctional isoharmonic and functional harmonic progressions. Psycho-acoustic experiment would be required to confirm or refute this.

Teaching

The inherent functional harmonic relationship between our twelve pitch classes are readily visualised on the Harmonic Terrain. Different scales and chord types have unique shapes on the Harmonic Terrain. Harmonics and sub-harmonics are clearly identifiable. The function cycle is from left to right so that the direction of harmonic progression is clear and function labels may be attached to each column if a tonic is identified. All these readily discernible aspects may be useful in the teaching of harmony.

Conclusion

The theory of harmony proposed is based on the inherent harmonic relationships between pure tones. The theory is based on the harmonic series, acoustic science and how our psycho-acoustic system appears to use harmonic to sub-harmonic assimilation to produce the sensation of a musical tone. This process of assimilation is proposed as the prototype for functional harmonic progression in music: all possible progressions can be explained by this. Harmony's richness can be seen as the articulation of the structure of the harmonic series in pitch class space.

Function emerges as a property of pitch class expressing a directional potential for harmonic progression in pitch class space. The function cycle is derived from the direction of harmonic to sub-harmonic assimilation inherent in the structure of our 12-tone music. Function is shown to be a consequence of the structure of our tonal system as derived from the harmonic series. Function and the function cycle are not inferred through empirical interpretation of harmonic syntax as composed; it is the harmonic syntax that follows the inherent structural function cycle. The Harmonic Terrain shows all possible harmonic relationships between the twelve chromatic pitch classes.

The present theory has been used to analyse some common, and not so common, progressions to illustrate that the function-cycle motif is always present. Apparent harmonic regression at the level of the score is explained as functional diaharmonic progression. That every musical tone can progress to any of the twelve can be explained using harmonic, diaharmonic and isoharmonic progressions. Chromatic pitches are consistently incorporated in the theory and not seen as special cases, they are part of one functional harmonic pitch class space. As a consequence of the theory it emerges that it is mistaken to assume that harmonic progression always takes place between the roots of chords, even when the roots may be clearly identified. Some routes for harmonic progression to a chord can be through musical tones that would not be identified as the root of a chord, and through pure tone harmonics below the musical surface as scored. A chord can be entered through a pitch with one function and left through a pitch with another, or the same, function. We hear and process much more beneath the score surface than we are consciously aware of.

References

Balsach, L. 2020. The fundamentals of harmonic tensions. Provisional translation to English Academia.edu.

Beament, J. 2005. How We Hear Music. Woodbridge: The Boydell Press. Cambridge: Cambridge University Press.

Bregman, A. S. 1990. Auditory Scene Analysis: The Perceptual Organization of Sound. Cambridge, MA: MIT Press.

Damschroder, D. 2008. Thinking About Harmony: Historical Perspectives on Analysis. Cambridge University Press.

Holland, S. 1989. Artificial Intelligence, Education and Music. PhD thesis published internally as OU IET CITE Report No. 88, July 1989.

Lendvai, E. 2015. Béla Bartók An Analysis of his Music. Kahn & Averill, London.

Krumhansl, C. L. and Kessler, E. J. (1982) Tracing the dynamic changes in perceived tonal organization in a spatial representation of musical keys. Psychological review 89, pp334-68.

Parncutt, R. 1989. Harmony: A Psychoacoustical Approach. Berlin: Springer-Verlag.

Parncutt, R., & Radovanovic, L. 2022. The missing fundamentals of harmonic theory: Chord roots and their ambiguity in arrangements of jazz standards. *Musicae Scientiae*, *27*(2), pp366-389. https://doi.org/10.1177/10298649211062934 (Original work published 2023)

Plomp, R. 1964. The Ear as a Frequency Analyzer, The Journal of the Acoustical Society of America, 36(9), pp1628–1636.

Rameau, J. 1971. *Treatise on Harmony*. Translated by Philip Gossett. New York: Dover Publications. Originally published as *Traité de l'harmonie* (Paris, 1722).

Rehding, A. 2008. Hugo Riemann and the Birth of Modern Musical Thought. New York: Cambridge University Press.

Schenker, H. 1980, p229. Harmony. Chicago: The University of Chicago Press.

Stumpf, C. 1911. Konsonanz und Konkordanz. *Beiträge zur Akustik und Musikwissenschaft*, *6*, pp116-150.

Terhardt, E. 1974. "Pitch, consonance, and harmony." Journal of the Acoustical Society of America, 55(5), pp1061–1069.

Terhardt, E. 2026. Topics of Research in Retrospect, Virtual Pitch. Harmony. terhardt.userweb.mwn.de/ter.html.