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Calibration of Serial Manipulators: Theory and Applications

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1. Introduction

The kinematic calibration is a procedure to improve the manipulator accuracy without mechanical means by acting on the manipulator controller.
Although manipulators are composed by accurate mechanical components, the precision of their motion is affected by many sources of error (Mooring et al, 1991). The final position accuracy is mainly influenced by: kinematic inaccuracy (due to manufacturing and assembly errors in both actuated and passive joints), load deformation (due to external forces including gravity) and thermal deformation (Reinhar et al, 2004). This is true for serial (Mooring et al, 1991) as for parallel (Wildenberg, 2000) manipulators as well. Each of these factors should be addressed with an appropriate compensation or calibration methodology. This work deals with kinematic inaccuracy, related to robot geometry, assembly and joint motion.
One possibility to compensate for geometrical errors is to perform a *kinematic calibration*. The robot is requested to reach some *desired* poses and the reached *actual* poses are measured. Then, the exact robot geometry is estimated analyzing the difference between the desired and the reached poses. This procedure requires a *parametric* identification of the manipulator which consists in the formulation of a geometrical model of the robot in which each source of error is represented by a parameter. The parameter set includes link lengths, joint axes inclination and joint coordinate offsets. The calibration consists in identifying the actual values of all these parameters. Once this operation is performed, it is possible to predict the pose error for any robot configuration and so it is possible to compensate for them by suitable joint motions.
The aim of this work is to address all the steps of the procedure which includes:

1. The development of a suitable kinematic model of a general serial manipulator;
2. One example of collection of experimental data;
3. The estimation of the numerical value of the manipulator parameters;
4. The error compensation.

For each phase different alternatives are described and critically compared.
A relevant part of the chapter summarises, compares and extends the existing techniques used to generate a suitable parametric kinematic model for a general serial manipulator. Rules for automatic generation of models with different characteristics are developed. These results will be used in a next chapter for the development of a parametric kinematic model for a general parallel manipulator (PKM).

An effective model for robot calibration must describe all the possible sources of error (it should be *Complete*) and, to avoid singularities, little geometrical errors must be described by small changes in the values of the corresponding parameters (*Parametrically Continuous* or *Proportional*). Such a model is often referred as CPC (Zhuang & Roth, 1992, Mooring et al, 1991). Furthermore, if each source of error can be described by one parameter only (absence of redundant parameters), the model is defined *Minimum* and this allows to obtain an unique numerical solution from the calibration process. In this paper such models are called MCPC (*Minimum Complete and Parametrically Continuous*).
For a given robot more than one valid MCPC model can be defined; generally speaking they are equivalent to each other, but each of them has different characteristics. Some comparisons about the different models are contained in the following Sections. Discussion about the accuracy achievable with the different models in presence of noise or measuring errors is outside the scope of this paper.
Being the aim of this work the analysis of geometrical errors, it will be assumed that the robot is composed by rigid links connected by 'ideal' joints (without backlash). All the possible sources of error will be considered constant. It will be also assumed that actuators are directly connected to the manipulator joints and so errors in the kinematics of the transmissions will be here neglected. These hypotheses are reasonable for many industrial manipulators.
In Sections 2 and 3 the basic concepts for the calibration procedure are presented. In Section 4 the general formula for the determination of the parameters number is discussed. Some different approaches used for serial robots are reordered (Sec.s 5 and 6), compared (Sec. 7) and a modified one is proposed (Sec. 8). After the explanation of an elimination procedure for the redundant parameters (Sec. 9) the calibration procedure for two different robots is discussed (Sec. 10 and 11). Eventually, Sec. 12 draws the conclusions.

J_i : i -th joint	$T(u,a)$: translation of a in $\vec{u}$ axis direction	$//$: parallel
R : revolute joint	$R(u,\varphi)$: rotation φ around axis $\vec{u}$	$\not\parallel$: not parallel
P : prismatic joint	$R(x,y,z,\alpha,\beta,\gamma)$: 3D rotation	$\perp$: orthogonal
R: number of revolute joints	Δa , Δb , Δc displacements along x, y, z	
P: number of prismatic joints	$\Delta\alpha$, $\Delta\beta$, $\Delta\gamma$ rotations around x, y, z	

Table 1. Symbols and abbreviations.

2. Methodological Bases

When choosing a parameter set to describe errors in a manipulator geometry, many different approaches can be followed; two of the most common are:

- Extended Denavit and Hartenberg approach ('ED&H');
- Incremental approach ('Inc').

When an 'ED&H' approach is adopted, a specific set of parameters is chosen to describe the robot structure (Denavit & Hartenberg, 1955) and errors are represented by variations of these parameters. The direct kinematics is represented as

$$S = F(Q, \Lambda_n + \Delta\Lambda) \tag{1}$$

where $S=[x,y,z,\alpha,\beta,\gamma]^t$ represents the gripper pose, $Q=[q_1,\dots q_n]^t$ is the vector of the joint coordinates, $\Lambda_n = [\lambda_1, \lambda_2, \dots, \lambda_N]^t$ is the vector of the nominal structural parameters and $\Delta\Lambda$ is

the vector of their errors.
In many cases Λ consists in the set of the Denavit & Hartenberg parameters. However, for calibration purposes, the classic D&H approach must be extended to assure the generation of a MCPC model in any situation (Sec. 5).
Conversely, if an 'Inc' approach is adopted, the nominal geometry of the robot is described by any parametric formulation $\overline{\Lambda}$ without requirements of minimality, completeness and proportionality. Errors are then represented by a suitable number of other parameters describing the difference between the nominal manipulator geometry and the actual one. This second set of parameters $\Delta\Lambda$ must be defined taking into account minimality, proportionality and singularity issues. There is no need that the number of the parameters in $\overline{\Lambda}$ equals that of those in $\Delta\Lambda$. In this case the direct kinematics is represented as

$$S = F(Q, \overline{\Lambda}, \Delta\Lambda) \tag{2}$$

3. Identifying the Parameter Values

Calibration can be defined as the procedure to estimate the numerical value of $\Delta\Lambda$ which better describes the kinematics of a given robot. It can be done using two different approaches: *Pose Measuring* and *Pose Matching* (Cleary, 1997).
Using the pose measuring approach, the robot is requested to reach a predefined '*desidered*' pose S_d and the calibration process is performed elaborating the difference between S_d and the '*real*' pose S_r reached by the gripper.
In the *pose matching* approach, the robot gripper is driven to a number of know poses and the corresponding joint rotations are measured. The difference between the expected joint coordinates and the actual ones is used as input for the identification procedure.

3.1 'Pose Measuring'

If we ask the robot gripper to move to a certain desired pose S_d, the gripper will reach the actual pose S_a:

$$\mathbf{S}_a = F(\mathbf{Q}_d, \Lambda_n + \Delta\Lambda) \quad \text{with} \quad \mathbf{Q}_d = G(\mathbf{S}_d, \Lambda_n) \tag{3}$$

where Q_d are the joint coordinates evaluated using the inverse kinematics model $G(\ldots)$, based on the nominal robot parameters Λ_n. The error in the gripper pose will be:

$$\Delta\mathbf{S} = \mathbf{S}_a - \mathbf{S}_d \tag{4}$$

Assuming that the magnitude of the parameter errors is sufficiently small, the equations can be linearized as:

$$\Delta\mathbf{S} \cong \mathbf{J}_\Lambda \cdot \Delta\Lambda \qquad \mathbf{J}_\Lambda = \frac{\partial \mathbf{F}}{\partial \Lambda} \tag{5}$$

where J_Λ is the jacobian matrix evaluated for $Q = Q_d$ and $\Lambda = \Lambda_n$.
If the value of ΔS can be measured for a sufficient number of poses, it is possible to estimate the value of $\Delta\Lambda$. The required equations are obtained rewriting Eq. (5) for each pose. A graphical representation of the procedure is presented in Figure 1.

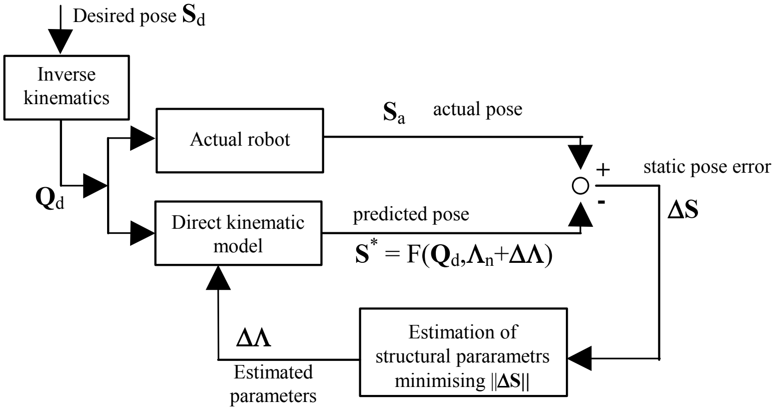

Fig. 1. Calibration of a robot using the Pose Measuring approach.

3.2 'Pose Matching'

When we force the robot gripper to reach a certain desired pose S_d, we predict the joint rotations Q_d using the inverse kinematics based on the nominal values Λ_n of the structural parameters:

$$\mathbf{Q}_d = G(\mathbf{S}_d, \Lambda_n) \tag{6}$$

However, the actual joints values Q_a are different from the predicted ones:

$$\mathbf{Q}_a = G(\mathbf{S}_d, \Lambda_n + \Delta\Lambda) \cong \mathbf{Q}_d + \frac{\partial G}{\partial \Lambda}\Delta\Lambda \tag{7}$$

or also:

$$\Delta Q = Q_a - Q_d \cong \frac{\partial G}{\partial \Lambda}\Delta\Lambda \tag{8}$$

$\Delta\Lambda$ is the vector containing the geometrical parameter errors and $\frac{\partial G}{\partial \Lambda}$ is evaluated for $S = S_d$ and $\Lambda = \Lambda_n$. Eq. (8) for pose matching is the equivalent of Eq. (5) for pose measuring. Since $\frac{\partial G}{\partial \Lambda}$ is generally not available, an alternative formulation can be used.

Linearizing the direct kinematics equation we have:

$$\mathbf{S}_a \cong F(\mathbf{Q}_d, \Lambda_n) + \frac{\partial F}{\partial \Lambda}\Delta\Lambda + \frac{\partial F}{\partial \mathbf{Q}}(\mathbf{Q}_a - \mathbf{Q}_d)$$

since S_a has been forced to be equal to $F(Q_d,\Lambda_n)$, and remembering Eq. (8) we get:

$$\frac{\partial F}{\partial \Lambda}\Delta\Lambda + \frac{\partial F}{\partial Q}\Delta Q \cong 0 \quad \Rightarrow \quad \frac{\partial G}{\partial \Lambda} = \left(\frac{\partial F}{\partial Q}\right)^{-1}\frac{\partial F}{\partial \Lambda} \tag{9}$$

The number of the geometrical parameters N is greater than the number of the joint parameters that can be measured for each gripper pose, but if the value of ΔQ can be measured for a sufficient number of poses, it is possible to estimate the value of $\Delta\Lambda$. The required equations are obtained rewriting Eq. (9) for all the poses. A graphical representation of the procedure is given in Figure 2.

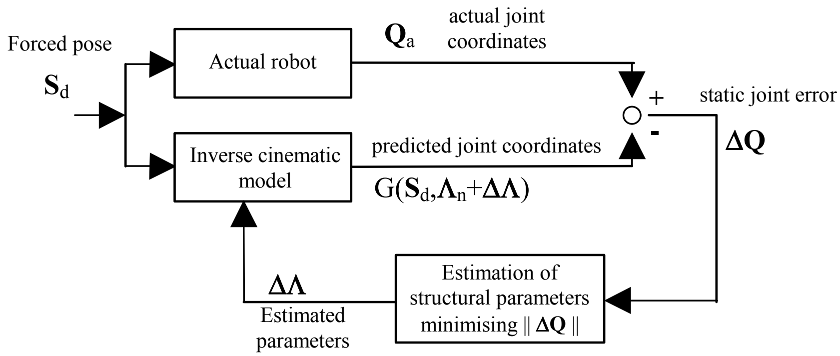

Fig. 2. Calibration of a robot using the Pose Matching Method.

3.3 Equivalence between Pose Matching and Pose Measured

An equivalence between the '*pose matching*' and the '*pose measuring*' can be established remembering that for a given value of the structural parameters we have:

$$\Delta \mathbf{S} \cong \frac{\partial F}{\partial \mathbf{Q}} \Delta \mathbf{Q}$$

As already stated in Section 3.1, the pose measuring approach for calibration is based on the difference between the predicted and the measured poses. The same procedure can be used any time that a set of corresponding pair of joint rotation Q and of gripper poses S are known for the actual robot.

In Section 3.1 the joint coordinates were forced to known values and the corresponding gripper pose was measured. An alternative approach is to force a known gripper pose and to measure the corresponding joint rotations.

Moreover, calibration programs written for the pose measuring approach can be used also for the pose matching case using the following guidelines.

The robot gripper is forced to a known pose $\overline{\mathbf{S}}$. The actual joint coordinates Q_a are measured. The theoretical gripper position S^* is predicted using the direct kinematics for $Q=Q_a$. Then, the robot calibration for pose measuring is performed assuming $\overline{\mathbf{S}}$ as measured pose S_a and $Q_d = Q_a$. A graphical representation of the resulting algorithm for pose matching is presented in Figure 3.

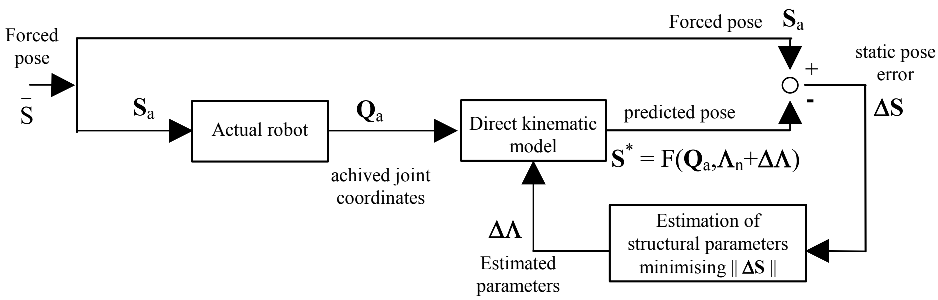

Fig. 3. Calibration of a robot using the Pose Matching Method (alternative approach).

3.4 Estimation of the Structural Parameters

Calibration Procedure: For the estimation of the structural parameters, three different identification procedures based on the guidelines presented in Sections 3.1 or 3.2 are here presented and compared:

- A non linear optimisation procedure;
- An iterative linearisation of the equations;
- An extended *Kalman* filter.

Their practical application will be presented in Section 11: a robot is driven to a set of known poses S_{ah} (h = *1, 2, ..., k*) and the corresponding joint rotation Q_{ah} are recorded. The mathematical procedures described in Sections 3.1 or 3.2 are then applied to estimate the structural parameters.

A non linear optimisation procedure ('amoeba'): The first method, experimented in this study to estimate the geometrical parameter errors, consists in writing Eq. (3) for a sufficient number of poses and in using a general purpose optimisation algorithm to find the value of $\Delta\Lambda$ which minimises the average error E_{op} based on the Euclidean norm:

$$E_{op} = \frac{1}{k}\sum_{h=1}^{k}\left\|\mathbf{S}_{ah} - F(\mathbf{Q}_{ah}, \Lambda_n + \Delta\Lambda)\right\|$$

where the subscript h is used to scan the k measured poses, S_{ah} is the *h-th* imposed gripper pose, Q_{ah} is the corresponding joint rotations, and *F(.)* is the predicted value for S. In the theoretical error free case, if the value of $\Delta\Lambda$ is exactly evaluated, E_{op} would be exactly null.
The algorithm, after several (many) iterations, gives an estimation of the value of $\Delta\Lambda$ which minimises E_{op}; E_{op} is also called residual.
A*moeba*, the optimization procedure we adopted, is adapted from (Flannery et al. 1992).

Iterative linearization of the equations ('linear'): The second method experimented consists in writing Eq. (5) for a sufficient number of poses and grouping all the equations in a linear system that can be solved to find $\Delta\Lambda$*:*

$$\mathbf{A} \cdot \Delta\Lambda = \mathbf{b} \quad \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_h \\ \vdots \\ \mathbf{A}_k \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_h \\ \vdots \\ \mathbf{b}_k \end{bmatrix} \tag{10}$$

Where A_h is J_Λ evaluated for the *h-th* pose and $b_h = \mathbf{S}_{ah} - F(\mathbf{Q}_{ah}, \Lambda_n)$.
The measures are generally redundant and so the system is solved with least squares criteria obtaining a first estimation for $\Delta\Lambda$ which is added to Λ_n obtaining a first prediction for the structural parameter values. This make possible a new better estimation of $b_h = \mathbf{S}_{ah} - F(\mathbf{Q}_{ah}, \Lambda_n)$ and a new solution of Eq. (10). The procedure can be iterated to improve the accuracy of the result. At each iteration j the last value of the parameters Λ_{j+1} replaces Λ_j:

$$\Lambda_{j+1} = \Lambda_j + \Delta\Lambda_j \quad \text{with} \quad \Lambda_0 = \Lambda_n$$

This procedure is repeated iteratively until the average error E_{it} reaches a stable minimum:

$$E_{it} = \frac{1}{k}\sqrt{\mathbf{b}^T\mathbf{b}}$$

E_{it} is also called residual.

An extended Kalman filter ('kalman'): A *Kalman* filter is a mathematical procedure to estimate the state of a system when uncertainties are present. Assuming that the vector of the geometrical parameters ΔΛ represents the state of a stationary system, an extended *Kalman* filter can be defined to give an estimation of ΔΛ starting from ΔS (Legnani and Trevelyan, 1996), (Clearly 1997).

The filter gives a new estimation of ΔΛ each time a new measure of ΔS is considered. The estimation $\Delta\Lambda_{h+1}$ of ΔΛ, after the *h-th* pose has been measured, is:

$$\begin{aligned} \Delta\Lambda_{h+1} &= \Delta\Lambda_h + \mathbf{M}_h\left(\mathbf{S}_{ah} - F\left(\mathbf{Q}_{ah}, \Lambda_n + \Delta\Lambda_h\right)\right) \\ \mathbf{M}_h &= \mathbf{P}_h\mathbf{C}_h^T\left(\mathbf{R} + \mathbf{C}_h\mathbf{P}_h\mathbf{C}_h^T\right)^{-1} \qquad \Delta\Lambda_0 = 0 \\ \mathbf{P}_{h+1} &= \left(\mathbf{1} - \mathbf{M}_h\mathbf{C}_h\right)\mathbf{P}_h \end{aligned}$$

Where C_h is the jacobian J_Λ evaluated in the *h-th* pose, M_h is the filter matrix gain after *h* steps, P_h is the matrix of the parameters covariance. R is the matrix of the measures covariance; extra diagonal elements in position *i, j* of matrices P (or R) indicates that the *i-th* and *j-th* parameters (or the *i-th* and the *j-th* measures) are correlated. P_0 representing the initial uncertainty of Λ should be initialised with suitable large values. The diagonal value of P contains a prediction of the accuracy of the estimation of ΔΛ, while R contains an estimation of the noise present in the measuring procedure. R and a series of S_{ah} and Q_{ah} must be given to the algorithm, which estimates ΔΛ and P.

After the processing of all the poses, an error index E_{ka} can be evaluated as

$$E_{ka} = \frac{1}{k}\sum_{h=1}^{k}\left\|\mathbf{S}_{ah} - F\left(\mathbf{Q}_{ah}, \Lambda_n + \Delta\Lambda\right)\right\|$$

4. Defining the Number of the Parameters

It has been proved for a n-DOF (degrees of freedom) serial manipulator (Mooring et al., 1991, Everett et al, 1987) that a model representing the pose of the gripper frame with respect to the fixed one, to be complete and minimum, must contain the following number of parameters:

$$N = 4R + 2P + 6 \tag{11}$$

being respectively R and P the number of revolute and of prismatic joints in the kinematic chain ($n = R + P$). This formula is derived under the hypothesis that

- Serial manipulators make use of *n* revolute or prismatic one-DOF joints (no spherical or cylindrical joints);
- All the joints are actuated (and so their motion is measured by the control system);
- The measures of all the 6 coordinates of the gripper are available for a number of different manipulator poses.

As stated in (Mooring et al, 1991), the proof of Eq. (11) is based on the observation that for revolute joints it is not relevant the position of the mechanical hinge but only the location of its geometrical axis which can be expressed in terms of two translations and two rotations (Fig. 4a). For prismatic joints only the axis direction is relevant and can be described with two rotations (Fig. 4b). At last, 6 parameters are necessary to define the gripper frame in an arbitrary location (Fig. 4c); this concept is directly applied in the 'Inc' approach (Sec. 6).
When only a partial measure of the gripper pose is available, the number of the identifiable parameters is reduced accordingly (Omodei et al, 2001):

$$N = 4R + 2P + G \tag{12}$$

being G the number of measurable coordinates of the gripper ($G \leq 6$). In milling applications, for example, the tool pose is identified by 5 coordinates, being the rotation about the tool axis redundant with the spindle motion.

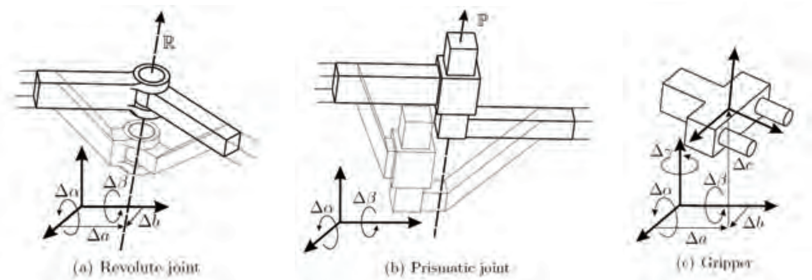

Fig. 4. The structural parameters necessary to define the location of: a) revolute joint axis (Δa , Δb , $\Delta\alpha$, $\Delta\beta$); b) prismatic joint axis ($\Delta\alpha$, $\Delta\beta$); c) gripper frame (Δa , Δb , Δc , $\Delta\alpha$, $\Delta\beta$, $\Delta\gamma$).

However, it is evident that 6 of the N parameters correspond to a wrong location of the robot base and they can be compensated simply by repositioning it. During experimental measures for the robot calibration, it is impossible to separate the effects of these errors with respect to errors in the location of the measuring instrumentation. Similarly the last 6 parameters describe the pose of the end effector with respect to the last link (i.e. the 'shape' and size of the gripper). So each time the end-effector is changed, these parameters change their value. We can conclude that only $N - 12$ parameters are intrinsically related to the manipulator structure. We call them *internal parameters*; their number is

$$N_i = N - 12 = 4R + 2P - 6 \tag{13}$$

The internal parameters can be then compensated during the manipulator construction or with a proper calibration procedure performed in the factory. The other 12, called *external parameters* , can be calibrated and compensated only in the user's working cell.

5. Extended D&H Approach (ED&H)

This methodology is based on an extension of that proposed by Denavit and Hartenberg for the kinematic description of mechanisms (Denavit & Hartenberg, 1955) and widely used for serial manipulators (Paul, 1981).

As well known, the links are numbered from 0 (the base) to n (the gripper). A reference frame is embedded on each link (base and gripper included) in such a way that the i-th joint axis coincides with the axis z_{i-1}. The pose of the i-th link with respect to the previous one is expressed by a 4×4 transformation matrix.

$$A_i = \left[\begin{array}{ccc|c} & R_i & & T_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

where R_i is a 3×3 rotation matrix and T_i is the vector of the origin coordinates of the i-th frame.

The direct kinematics of the manipulator can be expressed as

$$M = A_0 A_1 A_2 \ldots A_i \ldots A_n \tag{14}$$

where M is the matrix describing the gripper pose with respect to the base frame, n is the number of DOF. A_0 is a constant matrix representing the location of the first joint with respect to the base frame.

When the axes z_{i-1} and z_i are not parallel to each other, standard Denavit and Hartenberg rules can be used (Fig. 5) to position reference frames:

- Axis z_i coincides with $i+1$-th joint axis;
- Axis x_i is orthogonal both to z_i and z_{i-1} and directed from z_{i-1} to z_i;
- Axis y_i is chosen in order to create a right frame.

The base frame and the origin of the gripper frame are freely located.

The four D&H parameters of the i-th link are:

- The distance h_i between axes x_{i-1} and x_i which is called link *offset*;
- The distance l_i between axes z_{i-1} and z_i which is called link *length*;
- The angle φ_i between z_{i-1} and z_i which is called *twist*;
- The angle θ_i between x_{i-1} and x_i which is called *rotation*.

The relative location of frame i with respect to frame $i-1$ is then

$$A_i = R(z,\theta_i)T(z,h_i)T(x,l_i)R(x,\varphi_i) \tag{15}$$

where $R(u,\phi)$ is a rotation of ϕ around axis u and $T(u,t)$ is a translation t along u.

It can be noted that l_i and φ_i represent *intrinsic* geometric properties of the link, while θ_i and h_i are the joint motion and the assembly condition with respect to the previous link. This is quite evident for revolute joints, and similar considerations can be made for prismatic ones. As well known, for prismatic joints 2 out of the 4 parameters are redundant (Paul, 1981).

The D&H approach is often considered a good standardized way to describe robot kinematics. However calibration requires MCPC models and for several manipulator structures Eq. (15) must be modified as described in the following.

When two consecutive joint axes are parallel to each other the parameter h_i is not univocally defined and can be freely assigned. However if a small geometrical error equivalent to a rotation $\Delta\beta_i$ around y_i axis occurs, the joints are no longer parallel and $h_i \cong l_i/\Delta\beta_i$. So, for small variation of $\pm\Delta\beta_i$ around 0 the value of h_i changes abruptly from $-\infty$ to ∞ and therefore the model is no longer proportional.

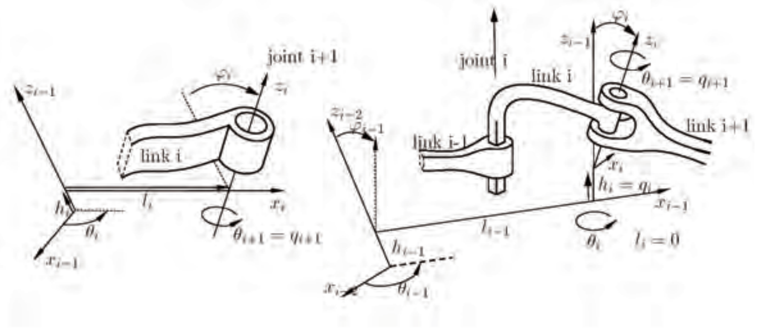

Fig. 5. Frames positions according to the Denavit and Hartenberg conventions. Case of i-th link with revolute (left) and prismatic joint (right).

In order to obtain a parametrically continuous model, when the i-th joint is revolute (Fig. 6a), the Hayati modification should be adopted (Hayati & Mirmirani, 1985)

$$A_i = R(z,\theta_i)T(x,l_i)R(x,\varphi_i)R(y,\beta_i) \tag{16}$$

while, when the i-th joint is prismatic (Fig. 6b), the **PR** modification is required:

$$A_i = T(z,l_i)T(y,h_i)R(x,\varphi_i)R(y,\beta_i). \tag{17}$$

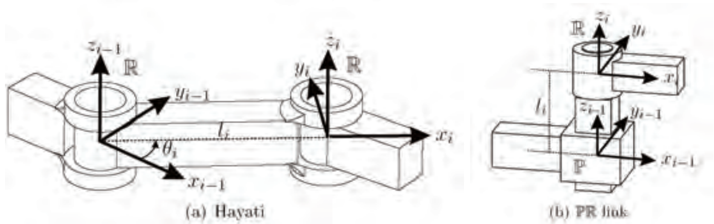

Fig. 6. Parallel joint axes: Hayati conventions for **RR** and **RP** (nearly)-parallel joint axes and the case of the **PR** link. In both cases the frames in the figures are represented for the nominal values of the rotations around x_i and y_i ($\varphi_i = \beta_i = 0$).

In both cases the nominal values of φ and β are zero, but they can be used to represent errors. In brief we can write

$$A_i = \begin{cases} R(z,\theta_i)T(z,h_i)T(x,l_i)R(x,\varphi_i) & \text{'D\&H'} \quad \textit{if } z_{i-1} \nparallel z_i \\ R(z,\theta_i)T(x,l_i)R(x,\varphi_i)R(y,\beta_i) & \text{'Hayati'} \quad \textit{if } z_{i-1} // z_i \textit{ link } \mathrm{RR} \textit{ or } \mathrm{RP} \\ T(z,l_i)T(y,h_i)R(x,\varphi_i)R(y,\beta_i) & \text{'PR'} \quad \textit{if } z_{i-1} // z_i \textit{ link } \mathrm{PR} \end{cases} \tag{18}$$

For prismatic joints, 4 parameters are used, however, to avoid redundancy, 2 of them suitably chosen for each specific robot are kept constant to their nominal value and eliminated from the calibration model (Mooring et al, 1991). The elimination process can be

performed using the algorithm described in Section 9.
Finally, in order to freely assign the gripper frame, the gripper matrix A_n must be generalized to contain 3 rotations and 3 translations. The expression of A_n depends on the i-th joint type J_n:

$$A_n = \begin{cases} R(z,\gamma_n)R(y,\beta_n)R(x,\alpha_n)\, T(z,c_n)T(y,b_n)T(x,a_n) & \text{'R6'} \quad \text{if } J_n = \mathbf{R} \\ T(z,c_n)T(y,b_n)T(x,a_n)\, R(z,\gamma_n)R(y,\beta_n)R(x,\alpha_n) & \text{'P6'} \quad \text{if } J_n = \mathbf{P} \end{cases} \tag{19}$$

The complete set of the geometrical parameters Λ is obtained by collecting all the variables used to describe the quoted matrices A_i. It is important to notice that some of the parameters coincide with the joints coordinates q_i:

$$\begin{array}{c|cc} & J_i = \mathbf{R} & J_i = \mathbf{P} \\ \hline i < n & q_i = \theta_i & q_i = h_i \\ i = n & q_i = \gamma_n & q_i = c_n \end{array} \tag{20}$$

6. Incremental Approach

When an incremental approach is adopted, Eq. (14) can be reformulated as

$$M = A_0 B_0 A_1 B_1 A_2 B_2 \ldots A_i B_i \ldots B_{n-1} A_n C \tag{21}$$

where matrices A_i describe the nominal geometry of the links and the joint motions while B_i and C describe geometric inaccuracy. Matrices A_i can be freely defined with the only constraint that z_i axis coincides with the $i-1$- th joint axis while B_i and C have the following form:

$$B_i = \begin{cases} R(x,\Delta\alpha_i)R(y,\Delta\beta_i)T(x,\Delta a_i)T(y,\Delta b_i) & \text{if } J_{i\text{-}1} = \mathbf{R} \\ R(x,\Delta\alpha_i)R(y,\Delta\beta_i) & \text{if } J_{i\text{-}1} = \mathbf{P} \end{cases}$$

$$C = R(x,\Delta\alpha_n)R(y,\Delta\beta_n)R(z,\Delta\gamma_n)\, T(x,\Delta a_n)T(y,\Delta b_n)T(y,\Delta c_n) \tag{22}$$

where Δa, Δb, Δc indicate translations respectively along x, y, and z axes and $\Delta\alpha$, $\Delta\beta$, and $\Delta\gamma$ describe rotations respectively around x, y, and z axes. Each matrix B_i represents the errors in the location of the axis z_i (i+1- th joint), while matrix C describes the errors in the gripper frame.
An alternative formulation for matrices B_i, proposed in (Zhuang & Roth, 1992) and (Zhuang et al 1992), is:

$$B_i(\Delta\theta_{xi}, \Delta\theta_{yi}, \Delta a_i, \Delta b_i) = \begin{bmatrix} 1 - \dfrac{\Delta\theta_{xi}^2}{1+\Delta\theta_{zi}} & -\dfrac{\Delta\theta_{xi}\Delta\theta_{yi}}{1+\Delta\theta_{zi}} & \Delta\theta_{xi} & \Delta a_i \\ -\dfrac{\Delta\theta_{xi}\Delta\theta_{yi}}{1+\Delta\theta_{zi}} & 1 - \dfrac{\Delta\theta_{yi}^2}{1+\Delta\theta_{zi}} & \Delta\theta_{yi} & \Delta b_i \\ -\Delta\theta_{xi} & -\Delta\theta_{yi} & \Delta\theta_{zi} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Delta\theta_{zi} = \sqrt{1 - \Delta\theta_{xi}^2 - \Delta\theta_{yi}^2} \tag{23}$$

For prismatic joints $\Delta a_i = \Delta b_i = 0$.

Comparing this definition of B_i with that of Eq. (22), for small errors, we get $\Delta\theta_{xi} \cong \Delta\beta_i$ and $\Delta\theta_{yi} \cong -\Delta\alpha_i$.

The set of the robot parameter errors $\Delta\Lambda$ is obtained collecting all the parameters (Δa_i, Δb_i, Δc_i, $\Delta\alpha_i$, $\Delta\beta_i$, and $\Delta\gamma_i$) of all the matrices B_i and C of Eq. (22). The offset errors Δq_i in the joint coordinates do not enter in vector $\Delta\Lambda$ because they are redundant with the above mentioned parameters.

7. Comparison Between ED&H and Incremental Approaches

The two approaches presented in Sections 5 and 6 have different characteristics. The development of the incremental parameter set can be more easily automated because the need to deal with different situations is minimized. The only test to be performed is to distinguish between revolute and prismatic joints (Eq. 22).

On the contrary, the extended D&H approach has the advantage that the offset errors in the joint coordinates q_i are explicitly present in the models. In some cases this could be an advantage because joint coordinate errors are responsible for a great percentage of the manipulator accuracy error and they can be easily compensated even in simple controllers without specific calibration software. However in the ED&H approach to avoid singularities and redundancies two cases must be dealt properly: consecutive parallel joint axes, and prismatic joints; this approach requires a more complicated algorithm.

An approach where the joint offset errors are explicitly present could be useful when extending the methodology to manipulators where many joints are not actuated (e.g. PKM) and so the corresponding joint parameters must be removed from the model.

A methodology which combines the good characteristics of the two approaches is developed in the next Section.

8. A Modified Incremental Approach

When an incremental approach is desired and the explicit presence of the joint offsets is requested, the following two-step procedure can be adopted. First of all Eq. (21) is modified introducing for each joint a matrix D_i describing the coordinate offset errors Δq_i:

$$M = A_0 B_0\ D_1 A_1 B_1\ D_2 A_2 \ldots D_n A_n\ C \tag{24}$$

where matrices D_i are defined as

$$D_i = \begin{cases} R(z, \Delta q_i) & \text{if } \mathrm{J_i} = \mathbf{R} \\ T(z, \Delta q_i) & \text{if } \mathrm{J_i} = \mathbf{P} \end{cases} \tag{25}$$

Secondly, Eq. (24) is analyzed in order to remove from matrices B_i all the terms redundant to the just introduced joint offset errors. This elimination process can be performed using differential analysis (infinitesimal motions) as described in Section 9.

After introducing matrices D_i and removing redundancy from the matrices B_i, the final number of the parameters does not change matching again the value predicted by Eq. (11).

9. Elimination of the Redundant Parameters

The algorithm described in this Section has been adapted from (Khalil et al, 1991, Khalil & Gautier, 1991, Meggiolaro & Dubowsky, 2000).

The variation $\Delta\lambda_i$ of the i-th geometrical parameter causes a variation ΔM_i of the matrix $M(Q,\Lambda)$ that describes the gripper pose $\Delta M_i = \frac{\partial M}{\partial \lambda_i}\Delta\lambda_i$. The correspondent roto-translation of the gripper $\Delta S_i'$ can be expressed in the base frame as:

$$\Delta S_i' = \Delta M_i M^{-1} = \frac{\partial M}{\partial \lambda_i} M^{-1}\Delta\lambda_i = L_i \Delta\lambda_i = \left[\begin{array}{ccc|c} 0 & -d\gamma_i & d\beta_i & dx_i \\ d\gamma_i & 0 & -d\alpha_i & dy_i \\ -d\beta_i & d\alpha_i & 0 & dz_i \\ \hline 0 & 0 & 0 & 0 \end{array}\right] \tag{26}$$

with (Legnani, Casolo et al, 1996)

$$\frac{\partial M}{\partial \lambda_i} = L_i M \quad \Leftrightarrow \quad \frac{\partial M}{\partial \lambda_i} M^{-1} = L_i \tag{27}$$

where dx_i, dy_i and dz_i are the translations and $d\alpha_i$, $d\beta_i$ and $d\gamma_i$ are the rotations of the gripper generated by the parameter error $\Delta\lambda_i$. The generic form of matrices L_i is

$$L_i = \left[\begin{array}{ccc|c} 0 & -\delta\gamma & \delta\beta & \delta x \\ \delta\gamma & 0 & -\delta\alpha & \delta y \\ -\delta\beta & \delta\alpha & 0 & \delta z \\ \hline 0 & 0 & 0 & 0 \end{array}\right] \tag{28}$$

which simplifies to ${}^rL$ for rotational parameters and to ${}^pL$ for translational ones

$${}^rL = \left[\begin{array}{ccc|c} 0 & -u_z & u_y & t_x \\ u_z & 0 & -u_x & t_y \\ -u_y & u_x & 0 & t_z \\ \hline 0 & 0 & 0 & 0 \end{array}\right] \qquad {}^pL = \left[\begin{array}{ccc|c} 0 & 0 & 0 & u_x \\ 0 & 0 & 0 & u_y \\ 0 & 0 & 0 & u_z \\ \hline 0 & 0 & 0 & 0 \end{array}\right] \tag{29}$$

where $U = [u_x, u_y, u_z]^t$ is the unit vector of the axis of motion (rotation or translation). Moreover for rotations $T = -U \times P = [t_x, t_y, t_z]^t$ where $P = [p_x, p_y, p_z]^t$ is one point of the axis around which the rotation is performed.

The relation between the error $\Delta S = [dx, dy, dz, d\alpha, d\beta, d\gamma]^t$ of the gripper pose and all the N errors in the manipulator geometry Λ can be expressed by means of the jacobian J_Λ

$$\Delta S = J_\Lambda \, \Delta\Lambda \tag{30}$$

The jacobian can be constructed transforming each matrix L_i into the i-th column of J_Λ

$$J_\Lambda = \left[\begin{array}{c|c|c|c|c} j_1 & \dots & j_i & \dots & j_N \end{array}\right] \tag{31}$$

with $j_i = [t_x, t_y, t_z, u_x, u_y, u_z]^t$ for rotational parameters and $j_i = [u_x, u_y, u_z, 0, 0, 0]^t$ for the translation ones. The i-th parameter is redundant if the i-th column of J_Λ is a linear combination of some other columns.

Generally, it is not necessary to construct the whole jacobian. For simplicity it is sometimes convenient to construct only the part relative to the links under analysis expressing all the terms in any suitable (adjacent) link frame.

For example, we consider two links of a robot (Fig. 7) with three revolute joints. The pose of the $i+1$-th frame with respect to the $i-1$-th one can be expressed as

$$\begin{aligned} M = \; & T(x_{i-1}, \Delta a_{i-1}) T(y_{i-1}, \Delta b_{i-1}) R(x_{i-1}, \Delta\alpha_{i-1}) R(y_{i-1}, \Delta\beta_{i-1}) R(z_{i-1}, q_i + \Delta q_i) T(x_{i-1}, l_i) R(x_{i-1}, \pi/2) \\ & T(x_i, \Delta a_i) T(y_i, \Delta b_i) R(x_i, \Delta\alpha_i) R(y_i, \Delta\beta_i) R(z_i, q_{i+1} + \Delta q_{i+1}) T(x_i, l_{i+1}) R(x_i, -\pi/2) \\ & T(x_{i+1}, \Delta a_{i+1}) T(y_{i+1}, \Delta b_{i+1}) R(x_{i+1}, \Delta\alpha_{i+1}) R(y_{i+1}, \Delta\beta_{i+1}) R(z_{i+1}, q_{i+2} + \Delta q_{i+2}) \end{aligned} \quad (32)$$

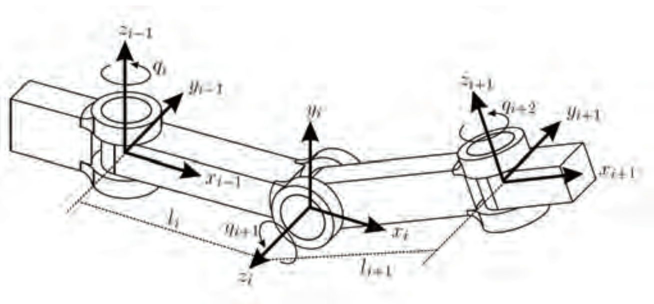

Fig. 7. Some links of a manipulator used to illustrate the elimination of the parameter redundant to Δq_i.

Suppose that we want to find the parameters redundant to Δq_i. For simplicity and according to Eq. (26) the jacobian is expressed in frame $i-1$ and so dx, dy and dz represent the displacement of one point embedded on frame $i+1$ which initially lies in the origin of frame $i-1$ (Legnani, Casolo et al, 1996). The relevant columns of J_Λ are presented in Table 2. The columns 5, 9, and 12 are linearly dependent to each other ($j_5 - j_9 - l_i j_{12} = 0$) and so one of the parameters Δq_i, $\Delta\beta_i$ or Δb_{i+1} must be eliminated from the model.

An alternative numerical approach for the elimination of the redundant parameters is presented in the example of Section 11. Another one is described in the chapter devoted to the calibration of PKM.

$J_\Lambda =$	$[j_1$	j_2	j_3	j_4	j_5^*	j_6	j_7	j_8	j_9^*	j_{10}	j_{11}	j_{12}^*	j_{13}	j_{14}	$j_{15}]$
	Δa_{i-1}	Δb_{i-1}	$\Delta \alpha_{i-1}$	$\Delta \beta_{i-1}$	Δq_i	Δa_i	Δb_i	$\Delta \alpha_i$	$\Delta \beta_i$	Δq_{i+1}	Δa_{i+1}	Δb_{i+1}	$\Delta \alpha_{i+1}$	$\Delta \beta_{i+1}$	Δq_{i+2}
dx	1	0	0	0	0	C_i	0	0	$l_i S_i$	0	$C_i C_{i+1}$	$-S_i$	...		
dy	0	1	0	0	0	S_i	0	0	$-l_i S_i$	0	$S_i C_{i+1}$	C_i	...		
dz	0	0	0	0	0	0	1	0	0	$-l_i$	S_{i+1}	0	...		
$d\alpha$	0	0	1	0	0	0	0	C_i	0	S_i	0	0	...		
$d\beta$	0	0	0	1	0	0	0	S_i	0	$-C_i$	0	0	...		
$d\gamma$	0	0	0	0	1	0	0	0	1	0	0	0	...		

Table 2. Relevant columns of the jacobian J_Λ of the manipulator of Figure 7 expressed in frame $i-1$, with: C_i=cos(q_i), S_i=sin(q_i), C_{i+1}=cos(q_{i+1}), S_{i+1}=sin(q_{i+1}).

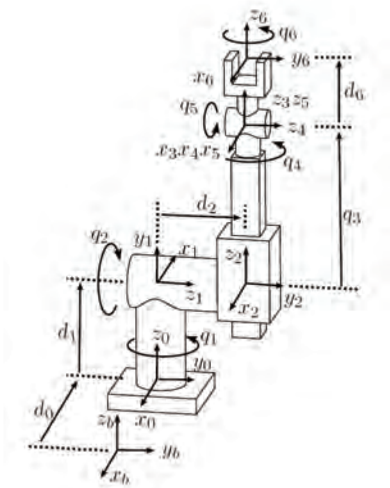

Fig. 8. The Stanford arm in its home position.

10. Example: Models of the Stanford Arm

The 6-dof Stanford arm (Fig. 8) is here analyzed as an example to construct the calibration parameters sets obtained applying the three quoted approaches.
The manipulator has 5 revolute joints and a prismatic one. The total number of parameters is $N = 6 + 4R + 2P = 6 + 4 \cdot 5 + 2 \cdot 1 = 28$.
Table 3 displays the parameter sets defined by the quoted approaches when external calibration is to be performed. The last two lines of the table displays the 12 external parameters to be removed from the model when internal calibration is required.
For the ED&H approach since the third link is prismatic, h_2 is redundant to h_3 and l_3 to h_4 and so two of them must be removed from the calibration model (in the example they are h_3 and h_4 indicated with parentheses in the table).
The modified incremental model is obtained from the incremental one observing that Δq_1 is redundant to $\Delta \beta_1$, Δq_2 is redundant to $\Delta \beta_2$, Δq_3 is redundant to Δb_4, Δq_4 is redundant to $\Delta \beta_4$, Δq_5 is redundant to $\Delta \beta_5$, and Δq_6 is redundant to $\Delta \gamma_6$.

Stanford Manipulator					
link	joint	type	Extended D&H nominal values	Incremental	Modified Incr.
Base 0	-	Hayati	$\theta_0 = 0$, $l_0=d_0$, $\varphi_0=0$, $\beta_0=0$	$\Delta\alpha_0$, $\Delta\beta_0$ Δa_0, Δb_0	$\Delta\alpha_0$, $\Delta\beta_0$, Δa_0, Δb_0
1	R	D&H	$\theta_1=q_1$, $h_1=d_1$, $l_1=0$, $\varphi_1=90^\circ$	$\Delta\alpha_1$, $\Delta\beta_1$, Δa_1 Δb_1	Δq_1, $\Delta\alpha_1$, Δa_1, Δb_1
2	R	D&H	$\theta_2=q_2$ $h_2=d_2$, $l_2=0$, $\varphi_2=90°$	$\Delta\alpha_2$, $\Delta\beta_2$	Δq_2, $\Delta\alpha_2$
3	P	PR	$l_3=q_3$, $(h_3=0)$, $\varphi_3=0$, $\beta_3=0$	$\Delta\alpha_3$, $\Delta\beta_3$, Δa_3, Δb_3	Δq_3, $\Delta\alpha_3$, $\Delta\beta_3$, Δa_3, Δb_3
4	R	D&H	$\theta_4=q_4$, $(h_4=0)$, $l_4=0$, $\varphi_4=-90^\circ$	$\Delta\alpha_4$, $\Delta\beta_4$ Δa_4, Δb_4	Δq_4, $\Delta\alpha_4$, Δa_4
5	R	D&H	$\theta_5=q_5$, $h_5=0$, $l_5=0$, $\varphi_5=90^\circ$	$\Delta\alpha_5$, $\Delta\beta_5$ Δa_5, Δb_5	Δq_5, $\Delta\alpha_5$, Δa_5, Δb_5
Gripper 6	R	R6	$\gamma_6=q_6$, $\beta_6=0$, $\alpha_6=0$, $c_6=d_6$, $b_6=0$, $a_6=0$	$\Delta\gamma_6$, $\Delta\beta_6$, $\Delta\alpha_6$, Δc_6, Δb_6, Δa_6	Δq_6, $\Delta\beta_6$, $\Delta\alpha_6$, Δc_6, Δb_6, Δa_6
External parameters base			$\theta_0=0$, $l_0=d_0$, $\varphi_0=0$, $\beta_0=0$, $h_1=d_1$, Δq_1	$\Delta\alpha_0$, $\Delta\beta_0$, Δa_0 Δb_0, Δb_1, $\Delta\beta_1$	$\Delta\alpha_0$, $\Delta\beta_0$, Δa_0 Δb_0, Δb_1, Δq_1
External parameters gripper			$\gamma_6=\Delta q_6$, $\beta_6=0$, $\alpha_6=0$ $c_6=d_6$, $b_6=0$, $a_6=0$	$\Delta\gamma_6$, $\Delta\beta_6$, $\Delta\alpha_6$ Δc_6, Δb_6, Δa_6	Δq_6, $\Delta\beta_6$, $\Delta\alpha_6$, Δc_6, Δb_6, Δa_6

Table 3. The 28 parameters of the 3 MCPC models of the Stanford manipulator (external calibration) and the 12 to be removed for internal calibration. For the ED&H approach, the values after the '=' sign are the nominal values, parameters in parentheses are redundant.

11. Example: Calibration of a Measuring Robot

11.1. Introduction

The above described calibration procedures have been applied to a measuring robot (Fig. 9) whose kinematics is described in Section 11.2. The manipulator is used in a shoe industry.

The robot is made by steel, the structure is quite stiff, its weight is partially compensated by compressed air pistons, it is not subjected to loads during operation. High precision incremental encoders are directly connected to the 5 revolute joints. For these reasons a pure kinematics procedure was considered appropriated.

The measuring robot is used to manually measure the shape of an object by touching it with the robot end-effector. The robot is requested to measure the position of the distal point of the end-effector and the direction of its axis.
The dime displayed in Figure 10 is used to force the manipulator to known poses for calibration purposes.

11.2 The parameters set

Eq. (11) would suggest the identification of 26 parameters for a complete calibration. However the end effector rotation around its axis is negligible, then a total of 25 structural parameters must be identified (Eq. (12) with G=5).
The 5 DOF robot under analysis has a structure similar to that of a PUMA robot. The direct kinematic problem was solved using a D&H procedure. An inverse kinematic solution was also derived in analytical form for $\Lambda = \Lambda_n$. The joint displacements are measured by high resolution encoders with a resolution of 0.018 degrees for step, that gives a gripper repeatability of about ±0.17 mm.

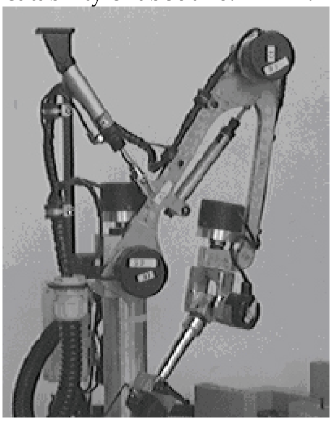

Fig. 9. The 5 DOF measuring robot.

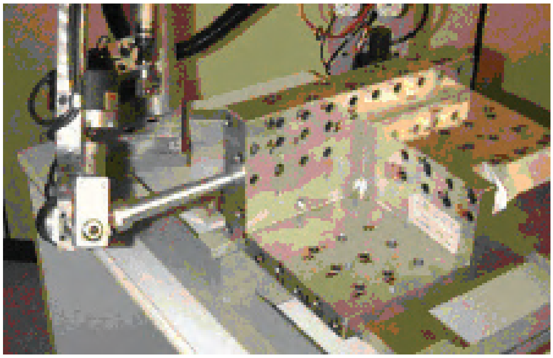

Fig. 10. Matching a pose using the dime.

Figure 11 shows a schematic view of the 5 DOF robot. Frame {0} is embedded on the robot base, while frames {1} to {5} are embedded on the corresponding link using the D&H convention (Denavit and Hartenberg, 1955) (axis z_i coincident with joint axis $i+1$).
The absolute reference frame {G} is the frame with respect to which each measure is taken. The nominal position of {G} is on the robot base with the z axis parallel to first joint axis, and it is coincident with frame {0}. z_0 is not parallel to z_G but its non-parallelism is caused by very small constant rotations δx_0 and δy_0 around x_G and y_G. The theoretical position of frame {0} with respect to {G} is $(X_0, Y_0, 0)^T$ and two position errors ΔX_0 and ΔY_0 are identified. Translations in the z direction are incorporate in the length of link 1.

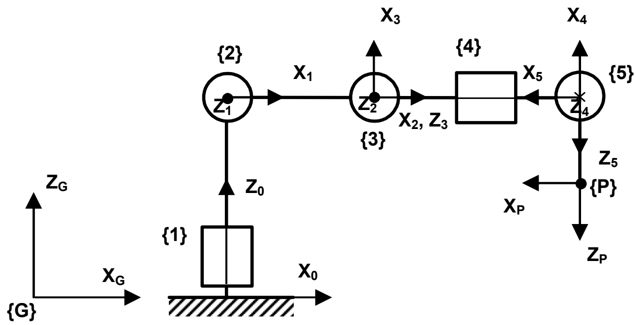

Fig. 11. Frame positions.

The pose of frame {0} with respect to {G} is then represented by A_{G0}:

$$A_0 = \begin{bmatrix} c_\psi & 0 & s_\psi & X \\ s_\chi \cdot s_\psi & c_\chi & -s_\chi \cdot c_\psi & Y \\ -c_\chi \cdot s_\psi & s_\chi & c_\chi \cdot c_\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{aligned} c_\alpha &= \cos(\alpha) \\ s_\alpha &= \sin(\alpha) \\ \chi &= \chi_0 + \delta\chi_0 \\ \psi &= \psi_0 + \delta\psi_0 \\ X &= X_0 + \Delta X_0 \\ Y &= Y_0 + \Delta Y_0 \end{aligned} \tag{33}$$

According to Eq. (15), with the exception of A_2, we get:

$$A_i = \begin{bmatrix} c_\vartheta & -s_\vartheta \cdot c_\alpha & s_\vartheta \cdot s_\alpha & a \cdot c_\vartheta \\ s_\vartheta & c_\vartheta \cdot c_\alpha & -c_\vartheta \cdot s_\alpha & a \cdot s_\vartheta \\ 0 & s_\alpha & c_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{aligned} \vartheta &= \vartheta_i + \delta\vartheta_i \\ \alpha &= \alpha_i + \delta\alpha_i \\ a &= a_i + \Delta a_i \\ d &= d_i + \Delta d_i \\ i &= 1,3,4,5 \end{aligned} \tag{34}$$

Since joint axes 2 and 3 are parallel, in order to avoid singularities in the calibration procedures, the transformation between frame {1} and {2} should be expressed accordingly, as described in Section 5. A suitable formulation is:

$$A_2 = R(z_1, \vartheta_2 + \delta\vartheta_2) \cdot T(z_1, d_1) \cdot T(x_2, a_2 + \Delta a_2) \cdot R(x_2, \alpha_2 + \delta\alpha_2) \cdot R(y_2, \psi_2 + \delta\psi_2)$$

We get:

$$A_2 = \begin{bmatrix} c_\vartheta \cdot c_y - s_\vartheta \cdot s_\alpha \cdot s_y & -s_\vartheta \cdot c_\alpha & c_\vartheta \cdot s_y + s_\vartheta \cdot s_\alpha \cdot c_y & a \cdot c_\vartheta \\ s_\vartheta \cdot c_y + c_\vartheta \cdot s_\alpha \cdot s_y & c_\vartheta \cdot c_\alpha & s_\vartheta \cdot s_y - c_\vartheta \cdot s_\alpha \cdot c_y & a \cdot s_\vartheta \\ -c_\alpha \cdot s_y & s_\alpha & c_\alpha \cdot c_y & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{aligned} \vartheta &= \vartheta_2 + \delta\vartheta_2 \\ \alpha &= \alpha_2 + \delta\alpha_2 \\ a &= a_2 + \Delta a_2 \\ d &= d_2 \\ y &= y_2 + \delta y_2 \end{aligned} \tag{35}$$

Finally, the pose of the gripper frame is:

$$A_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & Z_P + \Delta Z_P \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{36}$$

The gripper pose is represented by a product of the position matrixes:

$$M = A_0 \cdot \prod_{i=1}^{6} A_i$$

Therefore the complete set of the parameter to be estimated is:

$$\begin{aligned} \Lambda = [\ & X_0 \quad Y_0 \quad \chi_0 \quad \psi_0 \quad \vartheta_1 \quad d_1 \quad a_1 \quad \alpha_1 \quad \vartheta_2 \quad a_2 \quad \alpha_2 \quad \psi_2 \\ & \vartheta_3 \quad d_3 \quad a_3 \quad \alpha_3 \quad \vartheta_4 \quad d_4 \quad a_4 \quad \alpha_4 \quad \vartheta_5 \quad d_5 \quad a_5 \quad \alpha_5 \quad Z_p \quad]^T \end{aligned} \tag{37}$$

And the nominal value of the parameters are:

$$\begin{aligned} \Lambda = [\ & 0 \quad 0 \quad 0° \quad 0° \quad 0° \quad 252. \quad 0 \quad 90° \quad 0° \quad 0 \quad 320. \quad 0° \\ & 90° \quad 0 \quad 0 \quad 90° \quad 0° \quad 330. \quad 0 \quad 90° \quad -90° \quad 0 \quad 0 \quad 90° \quad 161. \quad]^T \end{aligned}$$

Length are given in millimetres and angles in degrees.

11.3 Results and Discussion

To verify the numerical optimization algorithms, as better described in (Omodei et al. 2001), we initially tested them with simulated measures created as follows. A number of joint rotations $Q^\#$ were chosen and the corresponding gripper poses $S^\#$ were evaluated after giving an arbitrary value to the structural parameter errors $\Delta\Lambda^\#$. The programs under test were asked to estimate the structural errors $\Delta\Lambda^\#$ assuming $Q_m = Q^\#$ and $S_m = S^\#$; $\Delta\Lambda^*$ should be identical to $\Delta\Lambda$. Before running the estimation procedure, the joint coordinates were corrupted adding random errors to simulate uncertainties in the measuring system. The convergence of the procedure and the achieved accuracy was estimated comparing S_d with $F(Q_d,\ \Lambda_n+\Delta\Lambda)$. The results show that if only geometric inaccuracy is present, it is possible to reach a robot accuracy close to the measuring error.
As a final step, experimental calibration was performed on the actual system (Fig.s 9 and 10). The measuring robot was forced to reach a set of predefined known poses and, for each of them, the corresponding joint rotation was measured.
A precision dime was manufactured by a CNC machine and holes created on different surfaces. The position and orientation of each hole is precisely known (it was measured by CMM machine). During the calibration procedure the operator inserts the gripper pin into the holes and collects the joint angles. These values are recorded together with the correspondent theoretic gripper pose position. In our case, for each pose we can measure the joint coordinates (5 data) and so we must repeat the measurement for a minimum of 25/5 = 5 poses. However to improve the calibration accuracy and to cover the whole working area with different gripper orientation the measuring dime was designed to have 72 insertion holes. Since some of the holes can be reached with two different robot configurations, so a total of 81 poses can be collected.

To estimate the robot repeatability, all the dime poses were recorded twice by two different operators recording the correspondent joint rotations. The gripper positions were evaluated with the nominal value of the parameters and the difference between the two sets was evaluated. The maximum difference was 0.176 mm, the average was $\bar{d}$ = 0.082 mm and its standard deviation was σ = 0.062 mm. Borrowing a definition from international standards (ISO 9283), we estimated the robot repeatability as $\bar{d} + 3\sigma = \pm 0.268$ mm.
The calibration procedure was performed as follows.
A total of 81 poses were collected (we call these data the *complete set*). The complete set was divided into two sub-sets: the *calibration set* and the *control set*. The calibration set was used as input for the program which evaluates the geometric parameters. The control set was used to verify the quality of the calibration.
In order to investigate the algorithm convergence, all the mathematical procedures were repeated 3 times using different sizes of the calibration and control set:

1. 40 calibration poses, 41 control poses;
2. 60 calibration poses, 21 control poses;
3. 81 calibration poses.

To verify the quality of the calibration, for each of the 81 poses we evaluated the distance between the known gripper positions with those estimated using the measured joint rotations and the evaluated structural parameter errors.
Table 4 contains the average residual, its maximum value and the standard deviation evaluated for all the situations considered. Considering the complete set the average position error before calibration procedure was about 4.702 mm with a standard deviation of 1.822 mm and 8.082 mm as maximum value.

	Calibration set (mm)				**Control set (mm)**			**Complete set (mm)**		
	Poses	average	s. dev.	max	average	s. dev.	max	average	s. dev.	max
a	40	4.789	1.784	8.082	4.616	1.854	7.572	4.702	1.822	8.082
b	60	4.692	1.785	7.574	4.732	1.921	8.082	4.702	1.822	8.082
c	81	4.702	1.822	8.082	-----	-----	-----	4.702	1.822	8.082

Table 4. Position errors (residual) before calibration procedure.

Table 5 presents the results of the calibration process performed using three algorithms and different number of poses. For each case we give the average position error (E_{op}, E_{it}, E_{ka}), their standard deviation and the maximum error.
The column labelled with 'time' represents the time necessary to process the data with a PC Pentium 100 MHz, highlighting the different efficiency of the algorithms.

		Calibration Set (Mm)				Control Set (Mm)			Complete Set (Mm)		
Alg.	Time	Poses	Average	S. Dev.	Max	Average	S. Dev.	Max	Average	S. Dev.	Max
amoeba	07′ 33″	40	0.310	0.196	0.894	0.301	0.178	0.775	0.305	0.187	0.865
	05′ 53″	60	0.399	0.261	1.127	0.461	0.192	0.828	0.415	0.246	1.117
	11′ 02″	81	0.316	0.249	1.069	-----	-----	-----	0.316	0.249	1.069
kalman	00′ 11″	40	6.004	4.579	11.69	5.431	4.597	11.81	5.714	4.597	11.81
	00′ 14″	60	6.676	3.048	11.54	6.931	2.611	13.99	6.742	2.945	13.99
	00′ 19″	81	5.767	2.675	11.85	-----	-----	-----	5.767	2.675	11.85
linear	00′ 06″	40	0.820	0.466	2.159	0.726	0.385	1.799	0.773	0.430	2.159
	00′ 08″	60	0.434	0.245	1.123	0.529	0.254	1.229	0.459	0.251	1.229
	00′ 13″	81	0.632	0.328	1.700	-----	-----	-----	0.632	0.328	1.700

Table 5. Comparison between calibration algorithms: residual error (25 parameters).

A comparison between the residual error between the calibration, the control and the complete set gives good information about the final results. As obvious the residuals in the control set are generally worst than those in the calibration set; the residuals of the complete set is an average of the two. However differences are small assuring that the number of the considered poses was sufficient to calibrate the robot on the considered working volume. This conclusion is supported by the fact that the results do not vary very much increasing the calibration set from 40 to 81 poses.

The results prove the effectiveness of the algorithms called '*amoeba*' and '*linear*' which reduce the gripper pose to about 1/8 of the initial value.

On the contrary the kalman filter gave worst results, though with other robots, this algorithm performed very well.

The robot is designed to operate only a subset of its working area and the calibration procedure cover just this part.

This means that some of the robot parameters could possibly be un-distinguible to others, because produce almost the same gripper pose error. This fact can reduce the effectiveness of the calibration algorithms which work badly with redundant parameters.

For these reason we decide to perform the robot calibration estimating just a reduced set of the parameter errors. We remember that complete convergence of all parameters is not necessary for a robot to be accurate within the calibrated area. However if the robot is operated out of the calibrated area, the accuracy will depend on how well each parameter has converged. The presented methodologies calculate a robot model, which best fits the measured data, which is not necessarily the correct model. We empirically selected the parameters that we consider more important for the robot accuracy. The choice of the parameters was based on a visual inspection of the robot structure. This subset (called the *reduced set*) contains the robot base position and orientation, the joint offsets, and the link length: only 14 of 25 parameters are considered

$$\begin{aligned}\Lambda^* = [\ & \Delta X_0 \quad \Delta Y_0 \quad \delta\chi_0 \quad \delta\psi_0 \quad \delta\vartheta_1 \quad \Delta d_1 \quad \delta\vartheta_2 \\ & \Delta a_2 \quad \delta\vartheta_3 \quad \delta\vartheta_4 \quad \Delta d_4 \quad \delta\vartheta_5 \quad \Delta d_5 \quad \Delta Z_p \quad]^T\end{aligned} \tag{38}$$

The algorithms supplied an estimation of the structural parameters Λ^*. These values were utilised to predict the pose of the gripper used during the measuring session. The difference between the estimated and the predicted pose positions are shown in Table 6.

		Calibration set (mm)				Control set (mm)			Complete set (mm)		
Alg.	Time	Poses	average	s. dev.	max	Average	s. dev.	Max	average	s. dev.	max
amoeba	02′ 11″	40	0.586	0.195	0.921	0.538	0.221	0.963	0.561	0.210	0.963
	03′ 52″	60	0.445	0.195	1.122	0.528	0.212	1.208	0.467	0.203	1.208
	07′ 38″	81	0.294	0.189	0.926	-----	-----	-----	0.294	0.189	0.926
kalman	00′ 04″	40	1.423	0.706	2.770	1.285	0.659	2.733	1.353	0.686	2.770
	00′ 07″	60	4.864	2.231	12.62	5.361	1.777	11.75	4.993	2.134	12.62
	00′ 11″	81	5.276	2.331	13.07	-----	-----	-----	5.276	2.331	13.07
linear	00′ 03″	40	0.711	0.214	1.256	0.669	0.214	1.129	0.690	0.215	1.254
	00′ 06″	60	0.691	0.271	1.462	0.784	0.258	1.289	0.715	0.270	1.463
	00′ 07″	81	0.680	0.266	1.485	-----	-----	-----	0.680	0.266	1.485

Table 6. Comparison between calibration algorithms: residual error (reduced set of 14 parameters).

The results are not very different from those obtained with the complete set of 25 parameters. In some cases they are even better. This mean that some redundant parameter was present obstructing the convergence of the calibration process.

As a final test we designed an improved calibration program which is able to decide which parameters should be important.

The program works in this way. Initially the program performs the calibration using the reduced set of parameters $\Delta\Lambda^*$ and the corresponding residual error E^* is evaluated. We indicate the number of parameters in the complete set as n_c, while only n_r of them are present in the reduced set. Then the calibration process is repeated n_c - n_r times considering at each time the reduced set plus one of the others additional parameters. For each of this estimation a new value of E_i^* is evaluated. The parameter which generate the lower value of E_i^* is selected. If E_i^* is significantly lower than E^*, the *i-th* parameter is added to the reduced set and the estimation procedure is repeated from beginning.

		Calibration set (mm)				**Control set (mm)**			**Complete set (mm)**		
Alg.	Time	Poses	average	s. dev.	max	Average	s. dev.	Max	average	s. dev.	max
amoeba	1h 09′	40	0.238	0.175	0.693	0.240	0.153	0.722	0.239	0.164	0.722
	1h 49′	60	0.209	0.183	0.939	0.250	0.207	0.977	0.219	0.190	0.977
	1h 44′	81	0.232	0.182	0.891	-----	-----	-----	0.232	0.182	0.891
kalman	02′ 20″	40	1.371	0.652	2.496	1.233	0.630	2.563	1.301	0.645	2.563
	04′ 43″	60	5.231	2.610	12.14	5.697	2.249	11.16	5.352	2.530	12.14
	07′ 21″	81	5.614	2.846	13.68	-----	-----	-----	5.614	2.846	13.68
linear	00′ 52″	40	0.711	0.214	1.256	0.669	0.214	1.129	0.690	0.215	1.256
	03′ 00″	60	0.469	0.185	0.896	0.536	0.183	1.022	0.487	0.187	1.022
	10′ 23″	81	0.409	0.185	0.991	-----	-----	-----	0.409	0.185	0.991

Table 7. Comparison between calibration algorithms: residual error (reduced set plus automatically selected parameters).

The iterative process is terminate when the inclusion of a new parameter improves the residual error less than 5%. Results obtained with this procedure are presented in Table 7.

Table 8 shows the parameters that have been added in the different trials; Δa_3 and $\delta\alpha_3$ seems more important because they have been selected more frequently by the algorithms.

Comparing Table 7 with Table 5 it is clear that the last version of the algorithm is slower but gives the best results producing lower residual errors. For example the '*amoeba*' algorithm the average residual on the complete set is reduced from 0.3 - 0.4 mm to about 0.2 mm. This means that the calibration procedure works better if the redundant parameters are removed.

		Added Parameters										
Algorithm	Poses	Δa_1	$\delta\alpha_1$	$\delta\alpha_2$	δy_2	Δd_3	Δa_3	$\delta\alpha_3$	Δa_4	$\delta\alpha_4$	Δa_5	$\delta\alpha_5$
amoeba	40	x					x	x				
	60	x		x	x		x	x	x			x
	81						x	x				
kalman	40					x						
	60								x			
	81					x						
linear	40											
	60					x		x				
	81	x				x	x	x	x	x	x	x

Table 8. Parameters automatically added to the reduce set Λ^*.

12. Conclusions

All the steps necessary to perform a kinematic calibration of serial manipulators have been discussed and applied to an actual manipulator.
This first part of the chapter has compared and extended two procedures for the identification of the geometrical parameters necessary to describe the inaccuracies in the kinematic structure of a *generic serial robot* . The discussion led to a new methodology called 'Modified Incremental' which holds the positive characteristics of the others. Each procedure generates a Minimum, Complete and Parametrically Continuous set of parameters.
First of all the formula for the determination of the total number of parameters has been discussed pointing out the distinction between *internal* and *external* parameters.
The D&H approach for the parameters identification has been extended to deal with all the special cases (adjacent parallel joint axes, prismatic joints and gripper frame). This approach has the good quality to include the joint offsets in the parameter set and shows that some parameters (l and φ) are *intrinsic* of the link while the others represent the joint motion and the assembly condition.
The Incremental approach is the most simple to be applied (only the type of the joints must be considered) and can be more easily automatized. The main drawback is that the joint offsets are not explicitly included in the parameters set. The Modified Incremental approach solves this problem and the proposed procedure for the elimination of the redundant parameters ensures the Minimality of the model.
The last part of this chapter fully describes a practical calibration of a measuring robot discussing different estimations algorithms and their performances.
All these concepts will be reviewed in the next chapter in order to be applied to a *generic parallel manipulator* .

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14. References

Cleary, W. (1997). Robot Calibration, Development and Testing of a Laser Measurement System, *Honours thesis*, The University of Western Australia, October 1997.

Denavit, J. & Hartenberg, R. S. (1955). Kinematic Modelling for Robot Calibration, *Trans. ASME Journal of Applyed Mechanics*, Vol. 22, June 1955, pp. 215 - 221.

Everett, L. J.; Driels, M. & Mooring B. W. (1987). *Proceedings of IEEE Int. Conf. on Robotics and Automation*, pp. 183-189, Vol. 1, Raleig NC, Mar. 1987.

Flannery B.P., Press W.H., Teukolsky S.A., Vetterling W.T., *Numerical Recipes in C, The Art of Scientific Computing*, Cambridge University Press, 1992.

Hayati, S. & Mirmirani, M. (1985). Improving the Absolute Positioning Accuracy of robot manipulators, *Journal of Robotic System*, Vol. 2, No. 4, page pp . 397-413.

International Standard ISO 9283 Manipulating industrial Robots_Performance criteria and related test methods.

Legnani, G. ; Casolo, F. ; Righettini, P. & Zappa B. (1996). A Homogeneous Matrix Approach to 3D Kinematics and Dynamics. Part 1: theory, Part 2: applications, *Mechanisms and Machine Theory*, Vol. 31, No. 5, pp. 573-605.

Legnani G., J. P. Trevelyan (1996), "Static Calibration of Industrial Manipulators: a Comparison Between Two Methologies", Robotics Towards 2000 27th Int. Symp on Industrial Robots, pp.111-116, 6-8 October 1996.

Khalil, W. ; Gautier, M. & Enguehard, C. (1991). Identifiable parameters and optimum configurations for robot calibration, *Robotica*, Vol.9, pp. 63-70.

Khalil, W. & Gautier, M. (1991). Calculation of the identifiable parameters for robot calibration, *Proceedings of IFAC 91*, pp. 888-892.

Meggiolaro, M. & Dubowsky, S. (2000). An Analytical Method to Eliminate the Redundant Parameters in Robot Calibration, *Proceedings of International Conference on Robotics and Automation (ICRA '2000)*, IEEE, pp. 3609-3615, San Francisco, CA, USA.

Mooring, B.W.; Roth, Z. S. & Driels, M. R. (1991). *Fundamentals of manipulator calibration*, John Wiley & Sons, Inc., USA.

Omodei, A. ; Legnani, G. & Adamini, R. (2001). Calibration of a Measuring Robot: Experimental Results on a 5 DOF Structure, *Journal of Robotic System* Vol.18 No.5, pp. 237-250.

Paul, R. (1981*). Robot Manipulators: Mathematics, Programming, and Control*, Mit Press, 1981.

Reinhar, G.; Zaeh, M. F. & Bongardt, T. (2004). Compensation of Thermal Errors at Industrial Robots, *Prod. Engineering* Vol. XI, No.1.

Wildenberg, F. (2000). Calibrations for Hexapode CMW, *Proceedings of* 2nd *Chemnitzer Parallel kinematik-Seminar: Working Accuracy of Parallel Kinematics*, pp. 101-112, Verlag Wissenschaftliche Scripten.

Wang, J. ; Masory, O. & Zhuang, H. (1993). On the Accuracy of a Stewart Platform-Part II: Kinematic Calibration and Compensation, *Proceedings of IEEE Int. Conf. on Robotics and Automation*, Atlanta May 1993.

Zhuang, H. & Roth, Z. S. (1992), Robot calibration using the CPC error model, *Robotics & Computer-Integrated Manufacturing*, Vol. 9, No. 3, pp. 227-237.

Zhuang, H. ; Roth, Z. S. & Hamano, F. (1992). A Complete and Parametrically Continuous Kinematic Model for Robot Manipulator, *IEEE Transactions on Robotics and Automation*, Vol. 8, No. 4, August 1992, pp. 451-463.